J Math Didakt (2023) 44:117–142
https://doi.org/10.1007/s13138-022-00206-w

ORIGINALARBEIT/ORIGINAL ARTICLE

Improving Prospective Teachers' Beliefs About a Double Discontinuity Between School Mathematics and University Mathematics

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Received: 23 July 2020 / Accepted: 19 April 2022 / Published online: 12 May 2022

Abstract Prospective teachers often perceive a "double discontinuity" between school mathematics and university mathematics. The first discontinuity can be described as the belief that there is no coherence between school mathematics and university mathematics, which forms part of the notoriously problematic transition from school to university. The second discontinuity can be described as a belief about the lack of relevance of university mathematics for the later professional practice of prospective teachers. Beliefs about coherence and relevance have been known to impact prospective mathematics teachers' interests and academic success. In this paper, we discuss an intervention involving 72 prospective secondary school mathematics teachers, aimed at influencing their beliefs about coherence and relevance. For this, we refer to the construct of beliefs as the main part of our theoretical framework, as well as the sub-constructs of beliefs regarding coherence and relevance. We then describe an intervention implemented within the first two years of mathematics courses, involving so-called "teacher-oriented" tasks that aim to trigger reflection on the benefit of university mathematics for teaching mathematics in school. The effect of the intervention was measured with a pretest-posttest experimental design using a questionnaire concerning teachers' beliefs about coherence and relevance. Our results show that the prospective teachers' beliefs about coherence and relevance generally decrease during the semester. However, statistically significant differences between the treatment group and a control group were found, especially regarding their beliefs about relevance.

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Keywords Double discontinuity · Beliefs · Coherence · Relevance · Utility value · Teacher education

Verringern der Überzeugungen von angehenden Lehrkräften zur doppelten Diskontinuität zwischen Schul- und Hochschulmathematik

Zusammenfassung Lehramtsstudierende nehmen häufig eine doppelte Diskontinuität zwischen Schulmathematik und Hochschulmathematik wahr. Die erste Diskontinuität kann als Überzeugung beschrieben werden, dass es keine Kohärenz zwischen Schulmathematik und Hochschulmathematik gibt, und ist damit Teil des bekanntermaßen problematischen Übergangs von der Schule zur Universität. Die zweite Diskontinuität lässt sich als Überzeugung über die mangelnde Relevanz der Hochschulmathematik für die spätere Berufspraxis angehender Lehrkräfte beschreiben. Es ist dabei bekannt, dass Überzeugungen über Kohärenz und Relevanz das Interesse und den akademischen Erfolg angehender Mathematiklehrkräfte beeinflussen. In diesem Artikel diskutieren wir eine Intervention für 72 Lehramtsstudierende der Sekundarstufe II, die darauf abzielt, ihre Überzeugungen über Kohärenz und Relevanz positiv zu beeinflussen. Dabei beziehen wir uns im theoretischen Rahmen auf das Konstrukt der Überzeugungen sowie als Teil dieses Konstrukts auf Überzeugungen zur Kohärenz und zur Relevanz. Anschließend beschreiben wir eine Intervention innerhalb der ersten zwei Jahre des Mathematikstudiums, die mit sogenannten Lehramtsaufgaben darauf abzielt, den Nutzen der Hochschulmathematik für den Mathematikunterricht in der Schule zu reflektieren. Die Wirkung der Intervention wurde anhand eines experimentellen Pre-Post-Test-Designs anhand eines Fragebogens zu den Überzeugungen der Lehrkräfte zur Kohärenz und zur Relevanz gemessen. Unsere Ergebnisse zeigen, dass die Überzeugungen der angehenden Lehrkräfte zu Kohärenz und Relevanz im Laufe des Semesters generell abnehmen. Allerdings fanden sich statistisch signifikante Unterschiede zwischen der Treatmentgruppe und einer Kontrollgruppe, insbesondere bei den Überzeugungen zur Relevanz.

Schlüsselwörter Doppelte Diskontinuität · Überzeugungen · Kohärenz · Relevanz · Utility value · Lehramtsausbildung

1 Introduction

Over 100 years ago, Klein (1908; 2016, p. 1) described the "double discontinuity" which prospective mathematics teachers experience as they progress from school to university and then from university back to school:

> The young university student found himself, at the outset, confronted with problems, which do not remember, in any particular, the things with which he had been concerned at school. Naturally he forgot these things quickly and thoroughly. When, after finishing his course of study, he becomes a teacher, he suddenly found himself expected to teach the traditional elementary mathemat-

ics accordingly to school practice; and, since he was scarcely able, unaided, to discern any connection between this task and his university mathematics, he will soon fell in with the time honoured way of teaching, and his university studies remain only a more or less pleasant memory which had no influence upon his teaching.

In a contemporary interpretation of the double discontinuity, Winsløw and Grønbæk (2014, p. 59) describe the first discontinuity as "the well-known problems of transition which students face as they enter university". Problems that students encounter with the transition from school to university are often considered in research on mathematics education (e.g., Gueudet 2008; de Guzmán et al. 1998; Thomas et al. 2015). Gueudet (2008, p. 244) used the term "rupture" to describe the difficult transition that includes fundamental shifts in ways of thinking, in the organisation of knowledge, in mathematical communication and, also, changes in methods of teaching. Whereas Gueudet (2008) mostly refers to knowledge aspects, Di Martino and Gregorio (2019) stated that "the mathematical tertiary transition is a complex phenomenon, and affective aspects and their relationship with cognitive aspects are surely part of this complexity". One affective part of the transition problem for students is described as "unfulfilled expectations" (Geisler and Rolka 2020, p. 603). These unfulfilled expectations influence students' beliefs about the gap between school mathematics and university mathematics (Hernandez-Martinez 2016). Di Martino and Gregorio (2019), following Niss (2003), used the term "coherence" or, respectively, "problem of coherence" to describe a connection (coherence) or a gap (incoherence) between school mathematics and university mathematics. The importance of an affective perspective on the "problem of coherence" is seen in the way students' beliefs about the coherence or incoherence of school mathematics and university mathematics can influence their achievement in the subject and also the rate of students dropping out (Di Martino and Gregorio 2019). Heublein (2014) also listed unfulfilled expectations as a reason for drop outs in mathematics-based studies. In this paper, we use the term "beliefs about coherence" to describe the belief of prospective mathematics teachers that there is coherence or incoherence between school mathematics and university mathematics.

Winsløw and Grønbæk (2014, p. 59) went on to call the second discontinuity "the (difficult) transposition of academic knowledge gained at university into relevant knowledge for a teacher". Although systematic research that investigates mathematics teachers' beliefs about this second discontinuity is scarce, there is some consensus that prospective teachers have difficulty understanding the value of university mathematics for their later career as mathematics teachers (e.g. Prediger 2013). From a broader perspective, the second discontinuity may be understood as a part of the construct of relevance that Priniski et al. (2018) described as "an individual's subjective perception of the degree to which a stimulus (an object, an activity, a topic) is connected (i.e., has some relation) to them personally". The construct of relevance overlaps with the construct of utility value (Albrecht and Karabenick 2018; Priniski et al. 2018), which is "determined by how well a task relates to current and future goals, such as career goals" (Eccles and Wigfield 2002, p. 120). Both relevance and utility value were found to impact a student's interest, academic success, and perseverance (Canning and Harackiewicz 2015; Nagengast et al. 2018; Neuville et al. 2007). Regarding teacher education, a failure to understand the relevance of university mathematics could have an adverse effect on the quality of a teacher's classroom practices (Allmendinger et al. 2013; White et al. 2006). In this paper, we use the term "beliefs about relevance" to describe prospective mathematics teachers' beliefs about the relevance of university mathematics for a teacher's classroom practice.

To conclude, the challenge of changing teachers' beliefs regarding the double discontinuity consisting of beliefs about coherence and relevance is well described in mathematics education research. Teachers' beliefs concerning both coherence and relevance seem to have an impact on prospective teachers' academic success and future classroom practice. Accordingly, there exist numerous approaches to change these teachers' beliefs, focusing on reflection about the relation between school mathematics and university mathematics (Artzt et al. 2012; Bauer and Partheil 2009; Isaev and Eichler 2017; Prediger 2013; Winsløw and Grønbæk 2014). The cited approaches are based on specific courses (capstone courses) or "authentic" tasks (Thanheiser et al. 2014) that are provided for prospective teachers. Some tasks in the courses emphasise those parts of school mathematics that are re-framed and elaborated in university mathematics (e.g., Bauer and Partheil 2009; coherence). Other tasks emphasise those specific situations of a teacher's professional practice ("jobs", cf. Bass and Ball 2004) which require a sophisticated knowledge of university mathematics (Prediger 2013; cf. also Isaev et al. in press; relevance). However, research results concerning the effect of these interventions for prospective mathematics teachers is sparse. For this reason, the main focus of this paper will be to measure the effect of an intervention aimed at changing prospective teachers' beliefs about coherence and relevance. Concerning prospective teachers, we restrict our research to secondary school teachers. Our main research question is as follows:

> What effect does an intervention—focusing on reflection on the relation between school mathematics and university mathematics—have on prospective secondary school mathematics teachers' beliefs about coherence and relevance?

In addition to tackling this main research question, this study also aims to contribute to existing findings concerning a correlation of beliefs about coherence and relevance with levels of interest, academic success and perseverance.

To this end, we first discuss our theoretical constructs, namely beliefs and the constructs of coherence and relevance. We go on to outline the constructs of interest, academic success, and perseverance as correlates of beliefs about coherence and about relevance. Afterwards, we explain briefly our project aimed at primarily changing teachers' beliefs about relevance and secondarily, beliefs about coherence. Based on this intervention, we describe our pretest-posttest experimental design for investigating the effect of our intervention. Finally, we report the results of our study, which demonstrated a significant impact of our intervention on prospective mathematics teachers' beliefs about coherence and relevance.

2 Theoretical Constructs

The main theoretical construct for researching prospective mathematics teachers' thoughts about a double discontinuity is the construct of teachers' beliefs (e.g., Philipp 2007). Based on this construct, we refer to beliefs about the double discontinuity, namely beliefs about coherence and relevance. Finally, we discuss existing findings on the impact of these beliefs on the levels of interest, academic success and perseverance of prospective teachers.

2.1 Teachers' Beliefs

Fives and Buehl (2012, p. 471) stated that "research on teachers' beliefs [...] runs the gamut of research methodologies, theoretical perspectives, and identification of specific beliefs about any number of topics." According to Fives and Buehl (2012), a universal definition of beliefs and a universal description of belief topics is impossible (cf. also Goldin et al. 2016; Pajares 1992). However, it is possible to outline specific aspects of teachers' beliefs that are crucial to our research approach. For our research approach, four aspects are important: the scope of a teacher's beliefs, the function of beliefs as a filter, the possibility to change beliefs, and the impact of beliefs.

Firstly, beliefs held by teachers can be defined as "psychologically held understandings, premises, or propositions" (Philipp 2007, p. 259) referring to specific subjects such as "(a) self, (b) context or environment, (c) content or knowledge, (d) specific teaching practices, (e) teaching approach, and (f) students" (Fives and Buehl 2012, p. 472). Fives and Buehl (2012, p. 472) describe teachers' beliefs concerning "content or knowledge" as encompassing the knowledge that "teachers learn themselves". Thus, prospective teachers develop beliefs about their learning.

Secondly, beliefs are understood to impact both how information about a subject is received and the actions taken in a specific situation (cf. Eichler and Erens 2015). Thus, beliefs "serve as a filter for information" (Fives and Buehl 2012, p. 478; cf. also Philipp 2007). Fives and Buehl (2012, p. 479) state that the function of beliefs as a filter is particularly relevant in teacher education since prospective teachers' beliefs "shape what and how they learn".

Furthermore, research showed that teachers' beliefs impact classroom practices (Davis et al. 2019; Eichler and Erens 2015; Skott 2015). Although there is research that has shown that the link between teachers' beliefs and classroom practice is unclear, Skott stated in his review that teachers' beliefs are "the default explanation for classroom practice" (Skott 2015, p. 22). In addition, there is evidence that teachers' beliefs and teachers' classroom practice have an impact on students' learning and students' beliefs (e.g., Eichler and Erens 2015; Staub and Stern 2002).

Finally, an important aspect of teachers' beliefs refers to the question of whether beliefs have a stable or dynamic character (Fives and Buehl 2012; Hannula 2012). Some research suggests beliefs are relatively stable human traits while other research shows changes in beliefs over time and after an intervention (Fives and Buehl 2012; Liljedahl et al. 2012). Based on the assumption that beliefs are difficult to change, there seems to be a consensus that belief change is possible only if a strong situational impact on existing beliefs occurs (cf. Liljedahl et al. 2012). Thus, teachers' beliefs may be formed by prospective teachers at university and—without a strong situational impact on existing beliefs—might remain relatively stable through a teacher's professional career and form the basis of their classroom practice. Research implies that the transition from school to university is a strong situational impact that changes students' beliefs (Geisler and Rolka 2020).

2.2 Coherence and Beliefs Concerning Coherence

The majority of studies on the "problem of coherence" (Niss 2003) between school mathematics and university mathematics focus on the content of university mathematics compared to the content of school mathematics (e.g. Clark and Lovric 2009; Gueudet 2008; de Guzmán et al. 1998; Thomas et al. 2015; Thomas and Klymchuk 2012; Winsløw and Grønbæk 2014). For example, the increased precision of mathematical language or the emphasis on abstract concepts or proofs are identified as challenges for students in the transition from school to university. Some researchers also consider university teachers and their perception of students' abilities to cope with the coherence problem in the transition from school to university (e.g., de Guzmán et al. 1998; Thomas et al. 2015; Thomas and Klymchuk 2012). For example, Thomas and Klymchuk (2012) concluded that university teachers put less emphasis on modelling or the practice of examples than school teachers do, but place more emphasis on conceptual thinking or rigour. Guzmán et al. (1998) reported that university teachers attribute lack of interest and lack of mathematical abilities to their students.

Only a few studies focus on students' beliefs about the transition from school to university with regard to the coherence problem. For example, the study of Guzmán et al. (1998) revealed that mathematics students miss the use of concrete examples or a specific textbook and are not used to an emphasis on proofs or "abstract developments". Geisler and Rolka (2020) researched the development of mathematics students' epistemological beliefs during the first year of study. They found that students' dynamic beliefs decrease during the transition from school to university. Since dynamic beliefs seem to adversely affect student dropout rates, Geisler and Rolka (2020) evaluate the development of students' beliefs as problematic. Hirst et al. (2004) investigated the development of beliefs of mathematics students during the first year of study, with a focus on beliefs about the difficulty of university mathematics or in their own ability to understand university mathematics as compared with school mathematics. They found increasing beliefs about difficulties with university mathematics and decreasing beliefs about their own ability to understand mathematics in university courses. Di Martino and Gregorio (2019) provided a qualitative study in which dropout students expressed their reasons for dropping out. Referring to the coherence problem, Di Martino and Gregorio (2019, p. 836) concluded that "the totality of dropout students claimed that the maths they found at university was far from high school mathematics (80.77%) or quite far (19.23%)".

2.3 Relevance and Beliefs Concerning Relevance

Relevance and utility value are motivational constructs (Albrecht and Karabenick 2018; Eccles 1983). "Utility value is determined by how well a task relates to current and future tasks" (Eccles and Wigfield 2002, p. 120). To describe the construct of utility value, Eccles (1983) uses the example of a prospective veterinarian who is required to take an advanced mathematics course and does not value mathematics but acknowledges the benefit of the discipline for his/her future career. Values, and specifically utility values, are understood as considerations about "desired endstates" (Wigfield and Eccles 1992, p. 281). Yeager et al. (2014, p. 560) further stated that a perceived utility value, which they called the purpose of learning, could "benefit the self" or could "have some effect on or connection to the world beyond the self". The construct of relevance is often understood as being synonymous with utility value (Albrecht and Karabenick 2018), although Priniski et al. (2018) see relevance also as an intrinsic part of motivation that is similar to the attainment value (cf. Eccles and Wigfield 2002). In research, the relevance of a task is often defined in a broader way, referring to subjects in university courses or academic learning (Acee and Weinstein 2010; Albrecht and Karabenick 2018). For this reason, we consider the construct of relevance and understand relevance —following Albrecht and Karabenick (2018)—as being synonymous with the construct of utility value. Following Yaeger et al. (2014) we make a distinction between an individual relevance (self) and a global relevance (beyond self). Also, we class motivation as a part of prospective teachers' mathematics related affect (Hannula 2012) and refer to the teachers' beliefs about relevance.

Mathematics-based studies, particularly mathematics as a service subject in engineering studies, is regarded from the perspective of interventions aimed at increasing relevance. Schmidt and Winsløw (2021) reviewed related research focusing on "authentic" tasks for engineering students, which reveal the relevance (or rather the utility value) of mathematics to cope with authentic engineering problems. The close connection of mathematics to challenges realistically faced by engineering students in their future professional practice is often emphasised in research concerning the transition from school to university (Flegg et al. 2012; de Guzmán et al. 1998; Wood et al. 2012). In research on prospective mathematics teachers' beliefs, the main focus is on the development of prospective teachers' epistemological beliefs and interventions aiming to change these beliefs (Cardetti and Truxaw 2014; Maasepp and Bobis 2014; Shilling-Traina and Stylianides 2013). However, there is limited research on changes to prospective mathematics teachers' beliefs about relevance. There are approaches referring to "capstone courses" for prospective teachers that aim to highlight the relevance of university mathematics for future teachers' professional careers (Artzt et al. 2012; Winsløw and Grønbæk 2014) including "authentic" tasks (Thanheiser et al. 2014). In this regard, Prediger (2013) defines an authentic task as one which addresses possible challenges in the professional lives of teachers, for example, "evaluating a textbook's approach to a topic" or "analysing and responding to student errors" (Bass and Ball 2004, p. 296). Also, there is a general discussion about the relevance of topics in teacher education programmes (Prediger et al. 2015). However, to the best of our knowledge, there is no research about the effect of such interventions on prospective teachers' beliefs about relevance.

2.4 Impact of Prospective Teachers' Beliefs About Coherence and Relevance

As outlined in the first section, a student's beliefs about relevance have been found to impact their level of interest (Canning and Harackiewicz 2015; Nagengast et al. 2018). Interest is a part of motivation in terms of intrinsic motivation (Eccles and Wigfield 2002). Interest could be understood as a student's development of an appreciation for a specific subject such as mathematics (Wild and Möller 2009). Interest has been found to impact academic success concerning grades, satisfaction levels and perseverance (Schiefele et al. 1993).

Along with performance, satisfaction is understood as part of students' academic success (Fleischer et al. 2019; Westermann and Heise 2018). One part of the construct of satisfaction is students' satisfaction with the content of the study (Westermann et al. 1996), such as mathematics or, rather, analysis and linear algebra for prospective mathematics teachers. In this study, we refer to students' satisfaction as a part of academic success and we hypothesise that prospective teachers' beliefs about relevance show a substantial correlation with student satisfaction.

Finally, students' beliefs about coherence and relevance have been shown to affect perseverance (Di Martino and Gregorio 2019; Heublein 2014; Neuville et al. 2007). Perseverance is understood as "retention, or the length of time a student remains enrolled at an institution" (Robbins et al. 2004, p. 262). Along with academic success in the form of performance or satisfaction, also perseverance is regarded as part of students' academic success (Fleischer et al. 2019; Robbins et al. 2004). A lack of perseverance is regarded as students' tendency to drop out or to change course and, thus, could be defined as the opposite of student satisfaction (Fleischer et al. 2019; Heublein 2014; Westermann et al. 1996).

3 Materials and Procedure

3.1 The Intervention Aiming to Change Teachers' Beliefs About Relevance (and Coherence)

The severe problems and obstacles associated with the transition from school mathematics to university mathematics—and which partly contribute to drop-out levels among prospective mathematics teachers'—are encountered at the beginning of university studies (Gueudet 2008; de Guzmán et al. 1998). For this reason, we set the intervention for prospective teachers in the first two years of the teacher education programme. Thus, we did not focus on a capstone course "with the aim of concluding or 'crowning' the experience, and to link academic competence and training with the needs of a professional occupation" (Winsløw and Grønbæk 2014, p. 63).

The syllabus for secondary school mathematics teacher education at the university where our study took place, is common for universities in Germany (Gildehaus et al. 2021; Liebendörfer 2018). The first two years have a strong emphasis on

In the subsequent task from a mathematics contest for students ("Känguru der Mathematik 2009"), the participants were asked to determine which of the following figures is the greatest.

(A) $\sqrt{2}-\sqrt{1}$ (B) $\sqrt{3}-\sqrt{2}$ (C) $\sqrt{4}-\sqrt{3}$ (D) $\sqrt{5}-\sqrt{4}$ (E) $\sqrt{6}-\sqrt{5}$

A student in grade 12 chose answer (E) and stated:

"$\sqrt{6}-\sqrt{5}$ is the greatest figure because roots are monotone. So, the greater x, the greater f(x). Thus, their difference is the greatest, as well (by going more to the right)."

1. Analyse the student's answer. Where do you see problems in the argumentation?

2. Provide your own student-oriented answer to this topic.

3. Show in general: $\lim_{n\to\infty} \sqrt{n} - \sqrt{n-1} = 0$

Fig. 1 Example of a teacher-oriented task referring to sequences and limits of sequences

mathematics courses and a lesser emphasis on mathematics education courses. The main mathematical subjects are linear algebra and analysis. The structure of the mathematics courses is also common across Germany: the courses consist of four hours of lectures a week and tutorials for two hours per week, in which the students work on exercises in small groups. Tasks that students must solve at home form the basis of the tutorials. To be permitted to take the final exam at the end of the semester, prospective teachers need to correctly resolve a specific number of these tasks. We integrated our intervention in the tutorials and homework tasks. For the homework tasks, we developed teacher-oriented tasks in addition to the usual mathematics homework tasks, which were compulsory for the prospective teachers.

The focus of the teacher-oriented tasks is on revealing the relevance of university mathematics for the future professional practice of prospective teachers. For this, we followed Prediger (2013) and designed the tasks to match possible jobs required from teachers in their professional practice (cf. Prediger 2013) and for which the teachers could employ specialised knowledge (Ball et al. 2008). Since we have already reported the development of these teacher-oriented tasks in a conceptual paper (Isaev and Eichler 2022), we will only briefly describe one example of the teacher-oriented tasks in this paper.

One topic of the course "Analysis I" consists of sequences and limits of sequences. The related tasks for exercises and homework refer to specific sequences and the computation of a limit, for example for the sequence $(a_n)_{n\in\mathbb{N}}$ with $a_n := \frac{2n^2+3n-7}{1-4n^2}$. Other tasks refer to using the definition of a limit of a sequence to prove a hypothetical limit of a specific sequence (a further example is given in an open repository, https://osf.io/tkg9w). The teacher-oriented task in this set of tasks is shown in Fig. 1.

According to a job analysis based on Ball et al. (2008), the teacher-oriented task in Fig. 1 calls on (prospective) teachers to evaluate the plausibility of students' responses and to give, or to evaluate, mathematical explanations. Other jobs which our teacher-oriented tasks refer to are, for example, evaluating a textbook's approach to a certain topic or choosing and using representations. Although we focused on revealing the relevance of university mathematics with regard to the prospective teachers' professional practice, it is possible to retrospectively estimate the coherence between school mathematics and university mathematics. For this reason, the primary focus of our intervention was changing prospective teachers' beliefs about relevance, but a secondary focus was also included, on changing prospective teachers' beliefs concerning coherence (cf. also, Isaev et al. in press).

As stated by Bauer and Partheil (2009) or Prediger (2013), simply postulating the relevance of university mathematics for professional practice does not increase the perceived benefit of university mathematics for prospective teachers. By contrast, a self-exploration of this relationship could be purposeful (cf. also Canning and Harackiewicz 2015). Our intervention, including the requirement to solve the teacher-oriented tasks in compulsory homework, follows the latter approach where prospective teachers themselves explore the relevance of the teacher-oriented tasks.

3.2 Research Question and Hypotheses

As outlined in the introduction, our main research question is as follows:

> What effect does an intervention—focusing on reflection on the relation between school mathematics and university mathematics—have on prospective secondary school mathematics teachers' beliefs about coherence and relevance?

Since our intervention and the teacher-oriented task primarily focused on prospective teachers' beliefs about relevance, our primary hypothesis refers to the effect of our treatment concerning these beliefs:

H_1: The treatment, including teacher-oriented tasks, influences the development of prospective mathematics teachers' beliefs about relevance.

In an exploratory way, we further analysed the effect of the treatment on the subscale of the scale concerning beliefs about relevance, i.e., the subscale concerning beliefs about individual relevance (self) and beliefs about general relevance (beyond self).

Our treatment focuses secondarily on prospective teachers' beliefs about coherence. Since the teacher-oriented task could also supply evidence about the coherence between school mathematics and university mathematics, we examined the following hypothesis concerning beliefs about coherence:

H_2 The treatment including teacher-oriented tasks influences the development of prospective mathematics teachers' beliefs about coherence.

Since existing research implies the impact of beliefs about relevance on these teachers' levels of interest in their studies (Schiefele et al. 1993), we formulated a related hypothesis regarding a correlation of beliefs about relevance and interest. This analysis may be viewed as a replication of existing findings that are, however, based on a new measurement of prospective teachers' beliefs. We further analyse the correlation of prospective teachers' beliefs about relevance and satisfaction (Westermann et al. 1996). Finally, beliefs about coherence and relevance were found to impact students' thoughts about dropping out, including a change of study programme, (Di Martino and Gregorio 2019) and perseverance (Neuville et al. 2007).

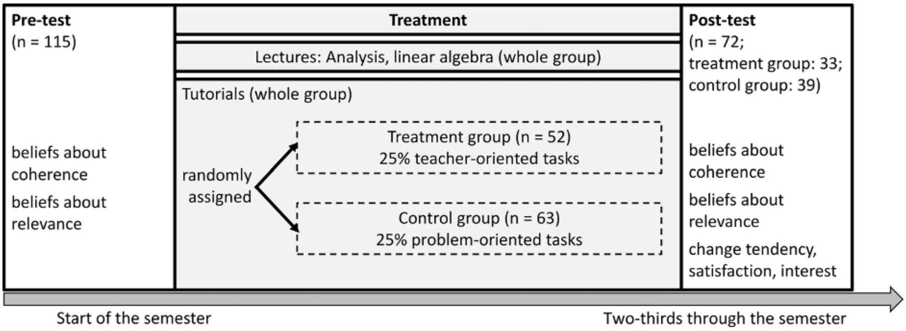


Fig. 2 Design of the study

We analysed this effect referring to the construct of change tendency (Kunter et al. 2010). The related hypotheses are as follows:

H_3: Teachers' beliefs about relevance are substantially correlated with prospective mathematics teachers' interest levels.

H_4: Teachers' beliefs about relevance are substantially correlated with prospective mathematics teachers' satisfaction with the teacher education programme.

H_5: Teachers' beliefs about coherence and about relevance are substantially correlated with prospective mathematics teachers' perseverance as measured by the teachers' tendency to change.

3.3 Design and Method of the Study

The sample in our study consists of 115 prospective mathematics teachers who were enrolled in two mathematics course, specifically in a linear algebra course (first semester) and an analysis course (third semester).

The design of our study (Fig. 2) had the following characteristics:

- The whole sample attended the courses (linear algebra, analysis) consisting of lectures and tutorials. The lectures were for four hours per week (analysis) and two hours in a week (linear algebra). The tutorials were two hours a week (analysis) and one hour a week (linear algebra). The prospective teachers were enrolled in the courses together with regular mathematics students. In the second week of the semester, all participants of the two lectures completed a pre-test consisting of a questionnaire designed to measure beliefs about coherence and relevance (cf. Sect. 3.4).
- The prospective teachers were randomly assigned to a treatment group ($n = 52$) and a control group ($n = 63$). The two groups differed regarding a tutorial for two hours a week and a homework assignment every week. The tutorials started after the pre-test.

- The second author conducted the tutorials for both groups and checked that the prospective teachers followed the assignment given to a group. All information about the tutorials was exclusively provided for one of the groups. However, an exchange of information about the different proceedings in both groups is possible.
- For the treatment group, the homework included traditional mathematical tasks and one teacher-oriented task as described above. In both courses, 25% of the tasks were teacher-oriented (one of four tasks in the analysis course every week and one of four tasks every two weeks in the linear algebra course).
- The homework for the control group included the same three mathematical exercises and an additional problem-oriented task with no special emphasis on the jobs of a teacher. During the tutorial, both the solutions for the traditional mathematical exercises and the additional task (teacher-oriented task, problem-oriented task) were discussed.
- The post-test was administered after about two-thirds of the semester and consisted again of the questionnaire aimed at measuring beliefs about coherence and relevance (cf. Sect. 3.4). 72 prospective teachers completed both the pre-test and the post-test. Due to missing items in the pre-test or the post-test, we analysed 68 prospective teachers concerning beliefs about coherence (treatment group $n = 33$; control group $n = 35$) and we analysed 66 prospective teachers concerning beliefs about relevance (treatment group $n = 32$; control group $n = 34$).
- In addition, the prospective teachers completed a questionnaire designed to measure students' study interest, satisfaction with their academic studies (satisfaction) and a tendency to change the field of study (change tendency).
- There was a total of 43 missing data in the post-test compared to the pre-test. We assume that the missing data represent students who dropped out from the course and from the field of study. Following Di Martino and Gregorio (2019), students who drop out seem to have specific characteristics, for example, concerning their evaluation of the coherence of school mathematics and university mathematics. For this reason, to avoid a bias, no imputation method was used to estimate missing data.
- The semester when the lectures and tutorials took place lasted 14 weeks. The students were required to solve 50% of homework tasks in order to receive permission to take the final written exam that was held directly after the semester.

The whole survey was voluntary and anonymity of the prospective teachers was guaranteed by the use of codenames. The study took place at a German university in the winter semester from October 2016 to February 2017.

3.4 Instruments

To measure beliefs about coherence and about relevance, we developed a questionnaire (cf. Isaev and Eichler 2017, 2021).

The questionnaire referring to (prospective) teachers' beliefs about coherence (double discontinuity—coherence: DD-C) consists of 8 items. One example of an item is as follows:

- In terms of content, university mathematics is related to school mathematics.

The questionnaire referring to (prospective) teachers' beliefs about relevance (double discontinuity—relevance: DD-R) consists of 10 items. Following Yeager et al. (2014), this part of the instrument includes two subscales. The first subscale, consisting of five items, refers to prospective teachers' beliefs about the individually perceived relevance of university mathematics with regard to one's own professional practice ("self"). One example of a related item is as follows:

- I will use university mathematics in many situations after finishing my university studies.

The second subscale, consisting of five items, contains items with a more general statement on the relevance of university mathematics for the professional practice "beyond the self" (Yeager et al. 2014, p. 560; "beyond self"):

- University mathematics is very useful in the professional practice of a teacher.

The questionnaires with all items can be found as an Appendix to this paper and in an open repository (https://osf.io/tkg9w).

We developed the questionnaires concerning beliefs about coherence and beliefs about relevance as Likert scales. For every Likert scale, we defined options to rate a statement from 1 (strongly disagree) to 6 (strongly agree). In a pilot study (cf. Isaev and Eichler 2017) we used an exploratory factor analysis for assigning items to a scale. The results imply the two scales described above, i.e., the scale concerning beliefs about coherence and the scale concerning beliefs about relevance. Although the results of the factor analysis did not imply the two subscales of the scale concerning beliefs about relevance, i.e., beliefs about individual relevance (self) and beliefs about global relevance (beyond self), we refer to the differentiation of the two subscales in an exploratory way based on the suggestion of Yaeger et al. (2014).

The reliability statistics using Cronbach's alpha are given in Table 1. The reliability of the two main scales and the two subscales concerning beliefs about relevance were (at least) very good. Due to the reliability level, we used sums for the two scales ranging from the minimum of 8 to the maximum of 48 for the scale DD-C and ranging from the minimum of 10 to the maximum of 60 for the scale DD-R. The two subscales of the scale DD-R, i.e., the subscale DD-R (self) and DD-R (beyond self) range from the minimum of 5 to the maximum of 30.

Our study was part of a larger project called PRONET (https://www.uni-kassel.de/themen/pronet/startseite.html). In this project, several variables of teachers' profes-

Table 1 Reliability of the scales concerning prospective teachers' beliefs in the pre-test and the post-test

	DD-C	DD-R	DD-R (self)	DD-R (beyond self)
Pre-test	$\alpha = 0.904$	$\alpha = 0.926$	$\alpha = 0.828$	$\alpha = 0.897$
Post-test	$\alpha = 0.890$	$\alpha = 0.927$	$\alpha = 0.864$	$\alpha = 0.872$

Table 2 Reliability of the scales concerning prospective teachers' beliefs in the pre-test and the post-test

	Interest	Satisfaction	Change tendency
Post-test	$\alpha = 0.872$	$\alpha = 0.860$	$\alpha = 0.865$

sional competence were measured in different sub-projects. Three of these variables refer to prospective teachers' interest, satisfaction and perseverance and thus to three variables that are potentially impacted by prospective teachers' beliefs about coherence and about relevance. First, we used a scale of Schiefele et al. (1993) consisting of 10 items (with slight adjustments), to measure prospective mathematics teachers' level of interest. Also, prospective teachers' satisfaction was measured by a scale developed by Westermann et al. (1996). From this scale, we used one subscale consisting of 3 items referring to prospective teachers' satisfaction with the content of the teacher education programme (cf. Westermann and Heise 2018) with slight adjustments. Finally, perseverance was measured by a scale developed by Kunter et al. (2010) referring to students' change tendency with 5 items with slight adjustments. For every Likert scale, we defined options to rate a statement from 1 (strongly disagree) to 6 (strongly agree). Every scale is provided in the Appendix and an open repository (https://osf.io/tkg9w).

The reliability statistics using Cronbach's alpha are given in Table 2. The reliability statistics were very good.

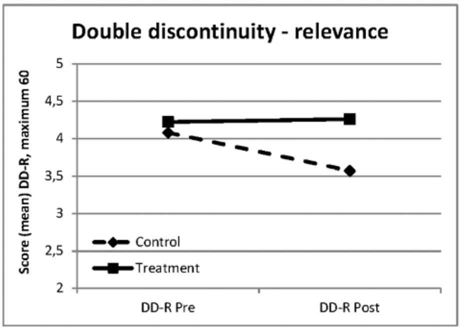
3.5 Data Analysis

The data were analysed with SPSS 27. One part of the analysis is based on a mixed ANOVA with repeated measurements (within-subjects: time; between-subjects: group). The Levene test for the variables DD-C and DD-R (and also DD-R self and DD-R beyond self) in the pre-test and post-test, for the treatment group and the control group, yielded no significant difference referring to the error variance. We further proved the normality for the mentioned variables. A Kolmogoroff-Smirnov test for normality yielded a significant result for the variables DD-R ($p = 0.014$) and DD-R self ($p < 0.001$). However, due to the robustness of the ANOVA against violation of normality (Schmider et al. 2010), we continued to use the ANOVA to analyse the effect of our treatment.

As well as an ANOVA, we used t-tests and correlation (Pearson's correlation coefficient) to analyse the data.

4 Results

As outlined in Sect. 3, the scale for measuring beliefs about relevance consists of 10 items. Each item provides six answer options from 1 (disagree) to 6 (strongly agree). Due to the very good reliability of the scale, we regard the sum for the 10 items with a minimum score of 10 and a maximum score of 60. In Fig. 3 and also the following figures, we show the mean score per item with the minimum of 1 and the maximum of 6 in the interval from 2 to 5. As the mean scores in Fig. 3



	Control	Treatment
Mean score (Pre)	4.1	4.2
Std.-Dev. (Pre)	0.91	0.72
Mean score (Post)	3.6	4.3
Std.-Dev. (Post)	0.86	0.77
n	34	32

Fig. 3 Development of the variable DD-R during the experiment in the treatment group and the control group

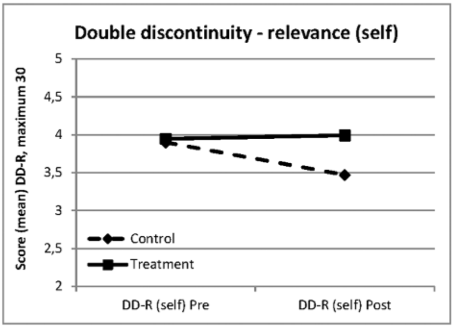
show, prospective teachers' beliefs about relevance decrease significantly without the treatment (t-test: $|t(33)| = 3.102$; $p = 0.004$; Cohens $|d| = 0.53$) and slightly increase in the treatment group (t-test: $|t(31)| = 0.291$; $p = 0.773$; Cohens $|d| = 0.05$).

A mixed ANOVA with repeated measures yield a statistically significant interaction effect of time * group, $F(1, 64) = 6.75$, $p = 0.012$, partial $\eta^2 = 0.095$ (medium effect). Thus, hypothesis H_1 is confirmed. In addition, there is a significant main effect of time on prospective teachers' beliefs about relevance, implying a decrease of these beliefs, $F(1, 64) = 5.03$, $p = 0.028$, partial $\eta^2 = 0.073$ (medium effect).

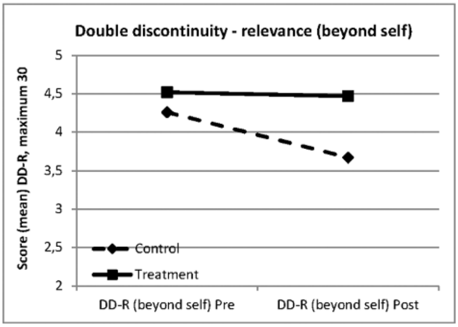
We also analysed the effect of the treatment on the sub-scales concerning DD-R, namely the prospective teachers' beliefs about individual relevance (DD-R self) and about global relevance (DD-R beyond self). The results concerning the two subscales are similar to the results concerning the main scale referring to beliefs about relevance (Fig. 4). In the control group, the beliefs about individual relevance (self) and about global relevance (beyond self) decrease significantly during the semester (t-test: DD-R self, $t(33) = 2.970$; $p = 0.006$; Cohens $|d| = 0.51$; DD-R beyond self, $t(33) = 2.971$; $p = 0.005$; Cohens $|d| = 0.50$). The related beliefs in the treatment group remain nearly constant (t-test: DD-R self, $|t(31)| = 0.337$; $p = 0.738$; Cohens $|d| = 0.06$; DD-R beyond self, $t(31) = 0.372$; $p = 0.712$; Cohens $|d| = 0.07$).

Concerning prospective teachers' beliefs about individual relevance, mixed ANOVA with repeated measures yield a statistically significant interaction effect of time * group, $F(1, 65) = 5.62$, $p = 0.021$, partial $\eta^2 = 0.079$ (medium effect). In this case, there is an almost significant main effect of time on prospective teachers' beliefs about individual relevance implying a decrease in these beliefs, $F(1, 65) = 3.62$, $p = 0.061$, partial $\eta^2 = 0.052$ (medium effect). Concerning prospective teachers' beliefs about global relevance, a mixed ANOVA with repeated measures yields a statistically significant interaction effect of time * group, $F(1, 65) = 4.83$, $p = 0.031$, partial $\eta^2 = 0.069$ (medium effect). In addition, there is a significant main effect of time on prospective teachers' beliefs about global relevance implying a decrease of these beliefs, $F(1, 65) = 6.83$, $p = 0.011$, partial $\eta^2 = 0.095$ (medium effect).

The scale for measuring beliefs about coherence consists of 8 items. Each item provides six answer options from 1 (strongly disagree) to 6 (strongly agree). Due



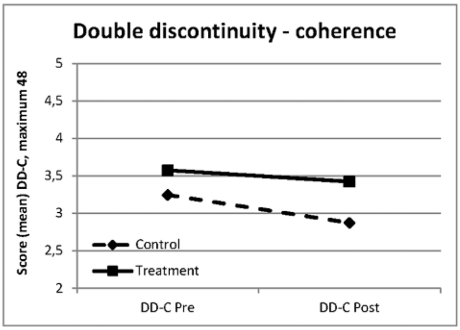
	Control	Treatment
Mean score (Pre)	3.9	4.0
Std.-Dev. (Pre)	0.94	0.72
Mean score (Post)	3.5	4.0
Std.-Dev. (Post)	0.85	0.79
N	34	32



	Control	Treatment
Mean score (Pre)	4.2	4.5
Std.-Dev. (Pre)	0.98	0.82
Mean score (Post)	3.6	4.5
Std.-Dev. (Post)	0.97	0.82
n	34	32

Fig. 4 Development of the variable DD-R self and DDR beyond self, during the experiment in the treatment group and the control group

to the very good reliability of the scale, we regard the sum for the 8 items with a minimum score of 8 and a maximum score of 48. In Fig. 5, we show the mean score per item with the minimum of 1 and the maximum of 6. As the mean scores in Fig. 5 show, prospective teachers' beliefs about coherence decrease significantly without the treatment (t-test: t(34) = 2.758; p = 0.009; Cohens |d| = 0.47). The beliefs about coherence also decrease in the treatment group, but the difference is not significant (t-test: t(32) = 1.086; p = 0.286; Cohens |d| = 0.19).



	Control	Treatment
Mean (Pre)	3.25	3.58
Std.-Dev. (Pre)	0.86	0.77
Mean (Post)	2.88	3.42
Std.-Dev. (Post)	0.84	0.73
n	35	33

Fig. 5 Development of the variable DD-C during the experiment in the treatment group and the control group

Table 3 Correlations among the prospective teachers' beliefs about relevance and coherence, and prospective teachers' level of interest, satisfaction and change tendency

	Double discontinuity relevance (DD-R)	Double discontinuity coherence (DD-C)
Study interest	0.418 [a]	0.426 [a]
Satisfaction	0.450 [a]	0.411 [a]
Change tendency	0.322 [b]	0.214

[a] indicates a correlation that is significantly different from 0 with $p < 0.01$
[b] indicates a correlation that is significantly different from 0 with $p < 0.05$

A mixed ANOVA with repeated measures does not yield a statistically significant interaction effect of time * group, $F(1, 66) = 1.28$, $p = 0.261$, partial $\eta^2 = 0.019$. Thus, hypothesis H_2 could not be confirmed. However, there is a significant main effect of time on prospective teachers' beliefs about coherence implying a decrease in these beliefs, $F(1, 66) = 7.28$, $p = 0.009$, partial $\eta^2 = 0.099$ (medium effect).

As outlined in Sect. 3, existing research highlighted an impact on students' beliefs about coherence and about relevance on interest levels, academic success and perseverance. For our study, we analysed the correlation between prospective teachers' beliefs about coherence and relevance, and the scales concerning prospective teachers' interest levels, satisfaction (academic success) and change tendency (perseverance; academic success). The results of these analyses are shown in Table 3.

The correlation analysis yielded significant results with a medium effect among the teachers' beliefs about relevance and about coherence on one hand and prospective teachers' study interest and prospective teachers' satisfaction on the other hand. Thus, hypothesis 3 and hypothesis 4 are confirmed.

In contrast, the correlation analysis did not yield a significant result for the correlation between prospective teachers' beliefs about coherence and these teachers' change tendency. However, the analysis yielded a significant result for the correlation between prospective teachers' beliefs about relevance and these teachers' change tendency (with a medium effect). For this reason, hypothesis 5 could only be partially confirmed.

5 Discussion and Conclusion

The focus of this paper was to analyse the effect of an intervention on prospective mathematics teachers' beliefs about coherence and about relevance. Beyond the specific effect of the treatment, it is striking that prospective teachers' beliefs decrease during the semester. In fact, our analysis identified a significant decrease (main effect) concerning time. With regard to teachers' beliefs about relevance, the decrease started at a high value. The average level of agreement with the relevance of university mathematics is about 4.2 on a scale from 1 to 6. Thus, prospective teachers seem to start their teacher education programme with the expectation that courses will be relevant in their later professional practice but then experience unfulfilled expectations during the courses. This result is in line with the findings of Geisler and Rolka (2020) concerning prospective teachers' beliefs. Moreover, our results are in line with several research results referring to decreasing parameter value for affective constructs such as students' satisfaction (Westermann et al. 2018), interest or motivation (Brahm and Jenert 2014; Eichler and Gradwohl 2021; Kyndt et al. 2015). Also, prospective teachers' beliefs about coherence show a decrease during the courses. Beliefs about coherence started with a lower value than beliefs about relevance. Our interpretation of this result is that prospective teachers have fewer expectations in relation to the coherence of school mathematics compared to the relevance of their mathematics study for their future professional practice. Beyond the specific perspective on beliefs about relevance and about coherence, our results contribute to the research on belief change (Liljedahl et al. 2012). Thus, the rupture (Gueudet 2008) that prospective teachers experience in the transition from school to university seems to be the strong situational impact that results in a considerable change of prospective teachers' beliefs.

The decrease of prospective mathematics teachers' beliefs is the basis for interpreting the results concerning the effect of the intervention. As hypothesised, our intervention showed a significant impact on the prospective teachers' beliefs about the relevance of university mathematics for their future professional practice. Our results imply that the intervention, including one of four tasks within a course lasting one semester, could considerably affect prospective mathematics teachers' beliefs. With regard to this effect, it is noteworthy that our intervention, compared to the control group, impedes a decrease of prospective teachers' beliefs about relevance. Our results contribute to research on changing university students' beliefs about relevance. First, our results provide a promising example of how "authentic tasks" aimed at revealing the benefit of university mathematics for the professional career of a teacher (c.f. Flegg et al. 2012; Schmidt and Winsløw 2021) may be designed for the specific group of prospective mathematics teachers. Furthermore, our results yield empirical evidence that teacher-oriented tasks which represent "jobs" to perform in subsequent professional practice (c.f. Prediger 2013) are appropriate in order to maintain a prospective teacher's belief about relevance whereas regular university courses seem to result in a decrease in these beliefs. In this regard, our results also contribute to research focusing on different ways of reflecting "on how what they are learning relates to their lives" (Nagengast et al. 2018, p. 105). In contrast to the results of Yeager et al. (2014), we did not find a substantial difference between prospective teachers' perceptions of the relevance of university mathematics for their own professional practice and from the professional practice of other teachers. Reasons for this result could be investigated further in future research.

Our intervention focused primarily on teachers' beliefs about relevance and to a lesser extent on teachers' beliefs about coherence (i.e., beliefs about a connection between school mathematics and university mathematics). Our results showed that the beliefs about coherence of these prospective teachers decreased in both groups—the treatment group that worked with the teacher-oriented tasks, and the control group. Moreover, the treatment had no statistically significant effect on the teachers' beliefs compared to the control group. Thus, although our teacher-oriented tasks could also trigger reflection on the relationship between the coherence of school mathematics and university mathematics, our results imply that a secondary focus of the teacher-oriented tasks is not enough to keep prospective teachers' beliefs about coherence stable or, rather, to reduce the prospective teachers' beliefs about unfulfilled expectations (c.f. Geisler and Rolka 2020). In our research, unfulfilled expectations are expressed in the items of our scale concerning teachers' beliefs about coherence. However, it could be an interesting question for further research to examine how teachers' beliefs about coherence measured by our scale relate to qualitative results about unfulfilled expectations as yielded, for example, in the research of Di Martino and Gregorio (2019). A further question is whether an intervention for prospective teachers may overcome their unfulfilled expectations. For example, the intervention of Bauer (2013) focuses on the prospective teachers' beliefs about coherence. Initial exploratory results imply that this specific focus may be enough to increase prospective teachers' beliefs about coherence (Isaev et al. in press). However, clear evidence based on an experimental setting is still to be found.

Contrary to our expectations, the correlation of prospective teachers' beliefs about coherence and relevance with their change tendency (perseverance) was only significant in relation to the beliefs about relevance. However, the significant correlations of prospective teachers' beliefs towards coherence and towards relevance and these teachers' study interest and satisfaction with the study is on one hand, a replication of existing research (Canning and Harackiewicz 2015; Nagengast et al. 2018; Neuville et al. 2007). On the other hand, our results provide these significant correlations for our specific instrument for measuring beliefs about coherence and relevance. In general, the correlation coefficients above 0.4 (satisfaction and study interest) show that prospective mathematics teachers' beliefs about coherence and relevance are considerably related to variables that represent academic success (satisfaction) or that seem to have an impact on academic success (interest; Fleischer et al. 2019; Westermann and Heise 2018). The missing, or rather, weaker correlations of prospective teachers' beliefs about coherence and about relevance, and teachers' change tendency, might represent a national characteristic of the teacher education programme. In Germany, the education programme for teachers of upper secondary schools includes (aside from pedagogy) two main topics. Thus, only one study topic of the teachers in our sample is mathematics. For this reason, a change tendency is dependent not only on the teacher's experiences in their study of mathematics but could also be impacted by the prospective teacher's experiences in studying the second topic.

The results of the correlation analysis have practical implications. Prospective teachers' beliefs about coherence and relevance correlate with their academic success. For this reason, it is a reasonable aim for teacher education programmes to increase, or at least to maintain, their beliefs about coherence and relevance. Since our intervention yielded promising results concerning stabilising prospective teachers' beliefs about relevance it seems to be possible to impact their beliefs and their academic success without a substantial change of existing teacher education programmes. However, what remains to be answered is whether an intervention that is more extensive compared to a single teacher-oriented tasks in tutorials for a mathematics lecture has an additional effect. A further question is how an intervention could be designed to maintain prospective teachers' beliefs about relevance and, in addition, improve prospective teachers unfulfilled expectations in terms of these teachers' beliefs.

As can be expected, our study has some limitations. Firstly, all results and interpretations are based on a small sample. A replication with a bigger sample would be valuable and would also allow us to make a differentiation between two mathematics courses such as linear algebra and analysis. Furthermore, we conducted the study in regular courses for analysis and linear algebra. As a result—although we randomly assigned the prospective teachers to groups and ensured they joined the group to which they were assigned—we cannot exclude the possibility that participants of different groups exchanged opinions about their experiences. However, the students did not have the option to move from one group to the second group. Our study could not provide results on the impact of prospective teachers' beliefs about coherence and relevance on their academic achievement in terms of mathematical performance (for example, as revealed by grades in final exams). Although we speculate that prospective teachers' beliefs impact academic achievement in the long run, this research question remains open. Finally, our study was in general not designed to investigate the effect of our intervention in the long term. For example, since prospective teachers for upper secondary school start their courses in mathematics education or internships after the introductory mathematics courses, it is at least possible that these educational courses or internships improve their beliefs about relevance and coherence. However, since prospective teachers' beliefs about university mathematics develop in the first two years, as our and other studies have shown (e.g., Geisler and Rolka 2020), there is some evidence that the usual design of the first mathematics courses for prospective teachers without "authentic" tasks revealing the relevance of university mathematics for the teachers' professional practice results in the double discontinuity that Klein (1908) described more than 100 years ago.

6 Appendix

Table 4 Questionnaire for measuring prospective mathematics teachers' beliefs towards a double discontinuity

German version DD-C:	*English translation DD-C:*
Die universitäre Mathematik hat inhaltlich kaum etwas mit der Schulmathematik zu tun	In terms of content, university mathematics and school mathematics are (rather) poorly linked
Es gibt in der universitären Mathematik zu wenig inhaltliche Bezüge zur Schule	In university mathematics, there are too few content-related links to school mathematics
Schulmathematik und universitäre Mathematik sind inhaltlich zwei verschiedene Welten	The subject matters of school mathematics and university mathematics are worlds apart
Es gibt viele inhaltliche Verbindungen zwischen Schulmathematik und der universitären Mathematik	There are many content-related links between university mathematics and school mathematics
Schulmathematik und universitäre Mathematik sind inhaltlich aufeinander bezogen	School mathematics and university mathematics are related to each other in terms of content
Die universitäre Mathematik weist viele inhaltliche Parallelen zur Schulmathematik auf	University mathematics shows many thematic parallels to school mathematics
Zwischen der Schulmathematik und der universitären Mathematik gibt es eine unüberwindbare Kluft	There is an insuperable gap between school mathematics and university mathematics
Die universitäre Mathematik hängt inhaltlich stark mit der Schulmathematik zusammen	University mathematics is strongly linked to school mathematics
German version DD-R:	*English translation DD-R:*
Die universitäre Mathematik ist sehr nützlich für den Lehrberuf	University mathematics is very useful for the teaching profession
Die universitäre Mathematik werde ich nach meinem Studium kaum wieder benötigen	I will hardly ever need university mathematics after studying
Durch die universitäre Mathematik werde ich gut auf das Berufsbild eines Mathematiklehrers vorbereitet	By learning mathematics at the university level, I am well prepared for the job of a mathematics teacher
Ohne die universitäre Mathematik könnte ich das Schulfach Mathematik kaum unterrichten	Without university mathematics, I could hardly teach mathematics in school
Das Lernen von Mathematik an der Universität ist nicht so wichtig für den Lehrerberuf	Learning mathematics at the university level is not very important for the teaching profession
Die universitäre Mathematik bringt mir nichts für meine spätere Tätigkeit als Lehrkraft in der Schule	I don't benefit from university mathematics with regard to my future occupation as a teacher at school
Das Lernen von Mathematik an der Universität ist im Hinblick auf Schule reine Zeitverschwendung	Learning mathematics at university is pointless with regard to school practice
Auf die Inhalte der universitären Mathematik werde ich nach meinem Studium noch häufiger zurückgreifen	I will frequently draw on the contents of university mathematics after having graduated
Die universitäre Mathematik bleibt nach meinem Studium eine Erinnerung, die auf meinen Unterricht keinen Einfluss hat	After my studies, university mathematics becomes a memory that does not impact my teaching practice
Die universitäre Mathematik ist nicht relevant für den Lehrerberuf	University mathematics is irrelevant for a teacher's professional practice

Table 5 Questionnaire for measuring prospective mathematics teachers' change tendency, satisfaction and interest

German version change tendency:	*English translation change tendency:*
Ich bin sicher, dass das Studienfach das Richtige für mich ist	I am sure that the subject is right for me
Für mich gibt es kein besseres Studienfach	For me there is no better subject
Wenn ich noch einmal wählen könnte, würde ich sofort wieder das Studienfach wählen	If I could choose again, I would immediately choose the subject again
Ich habe schon öfter einen Studienfachwechsel in Erwägung gezogen	I have often considered changing subjects
An der Entscheidung für das Studienfach habe ich noch nie gezweifelt	I have never doubted the decision to study the subject
German version satisfaction:	*English translation satisfaction:*
Ich habe richtig Freude an dem Fach, das ich studiere	I really enjoy the subject I'm studying
Insgesamt bin ich mit meinem jetzigen Studienfach zufrieden	Overall, I am satisfied with my current subject
Ich finde mein Studienfach wirklich interessant	I find my subject really interesting
German version interest:	*English translation change interest:*
Die Beschäftigung mit den Inhalten aus der universitären Mathematik gehört nicht gerade zu meinen Lieblingstätigkeiten	Dealing with the contents and problems of university mathematics is not exactly one of my favourite activities
Über Inhalte aus der universitären Mathematik zu reden, macht mir nur selten Spaß	I rarely enjoy talking about the content of university mathematics
Die Beschäftigung mit bestimmten Inhalten aus der universitären Mathematik wirkt sich positiv auf meine Stimmung aus	Being engaged with specific content of university mathematics has a positive effect on my mood
Auch wenn das Mathematikstudium anstrengend ist, so ist die Beschäftigung damit doch eine schöne Sache	Even if studying mathematics is exhausting, it is still a nice thing to do
Wenn ich ehrlich sein soll, ist mir die universitäre Mathematik manchmal eher gleichgültig	To be honest, I sometimes don't care about university mathematics
Die Beschäftigung mit bestimmten Inhalten aus der universitären Mathematik ist mir wichtiger als Zerstreuung, Freizeit und Unterhaltung	Dealing with certain course content of university mathematics is more important to me than a diversion, leisure and entertainment
Ich bin sicher, dass das Mathematikstudium meine Persönlichkeit positiv beeinflusst	I am sure that my study of mathematics will have a positive influence on my personality
Wenn ich genügend Zeit hätte, würde ich mich mit bestimmten Fragen der universitären Mathematik, auch unabhängig von Prüfungsanforderungen, intensiver beschäftigen	If I had enough time, I would deal more intensively with certain questions of university mathematics, regardless of the examination requirements
In meiner Freizeit beschäftige ich mich nur ungern mit Problemen der universitären Mathematik	In my free time, I don't like to deal with problems related to university mathematics
Ohne äußeren Druck würde ich mich wohl nicht so regelmäßig mit Inhalten aus der universitären Mathematik beschäftigen	Without external pressure, I would probably not deal so regularly with content in university mathematics

Acknowledgements "This project is part of the 'Qualitätsoffensive Lehrerbildung', a joint initiative of the Federal Government and the Länder which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research. The authors are responsible for the content of this publication."

Funding Open Access funding enabled and organized by Projekt DEAL.

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