The Economic Impact of Digital Payment Systems

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Summary

Private digital payment systems, e.g., Bitcoin or Ethereum, allow for transactions without the need for a financial institution. These institutions, central and retail banks, may thus observe a decline in the demand for their own payment systems, i.e., cash and deposits. Several questions arise: do digital payment systems affect the value of analog currencies such as cash? Are digital payment systems able to increase welfare? On the other hand, are central banks able to control providers of private digital payment systems? Do retail banks survive if central banks issue their own digital currency? To answer these questions, monetary search models are extended by digital money. Even if there are several studies which extend monetary search models by a secondary currency, the currency competition and the welfare effects are less well explored. It turns out that digital currencies reduce the value of analog currencies such as cash as long as digital currencies have a positive return. Nevertheless, digital currencies are able to increase welfare if the share of users is limited. In addition, central banks are able to tilt the playing field by providing an interest-bearing central bank digital currency. In equilibrium, retail banks face a lower profit but survive. Providers of private digital payment systems such as miners, on the other hand, go bankrupt.

Zusammenfassung

Private digitale Zahlungsmittel wie Bitcoin oder Ethereum ermöglichen Transaktionen ohne die Notwendigkeit eines Intermediärs. Jene Intermediäre, Zentralbanken und Geschäftsbanken, könnten daher einen Rückgang der Nachfrage nach ihren eigenen Zahlungsmitteln (Bargeld und Einlagen) verzeichnen. In diesem Kontext entstehen zahlreiche Fragen: Beeinflussen neue digitale Zahlungsmittel den Wert traditioneller analoger Zahlungsmittel wie Bargeld? Können digitale Zahlungsmittel die Wohlfahrt erhöhen? Sind Zentralbanken in der Lage, Anbieter privater digitaler Zahlungsmittel zu kontrollieren? Bleiben Geschäftsbanken zahlungsfähig, wenn Zentralbanken eine eigene digitale Währung anbieten? Um jene Fragen zu beantworten, werden monetäre Suchmodelle um digitales Geld erweitert. Obwohl bereits zahlreiche Arbeiten existieren, die monetäre Suchmodelle um eine zweite Währung erweitern, so sind die Erkenntnisse zum Währungswettbewerb und den Wohlfahrtseffekten begrenzt. Es zeigt sich, dass digitale Zahlungsmittel den Wert von analogen Zahlungsmitteln wie Bargeld mindern, sofern digitale Zahlungsmittel eine positive Rendite aufweisen. Dennoch können digitale Zahlungsmittel die Wohlfahrt erhöhen, solange der Anteil an Nutzern begrenzt ist. Darüber hinaus sind Zentralbanken in der Lage, den Zahlungsverkehr zu kontrollieren, indem sie eine verzinsliche digitale Zentralbankwährung emittieren. Geschäftsbanken bleiben zahlungsfähig, verzeichnen jedoch geringere Gewinne. Anbieter privater digitaler Zahlungssysteme gehen insolvent.

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Preface

After four years of research, this work marks the end of my PhD studies at the University of Kassel. However, it can by no means to be classified as an individual achievement. Without the support of several people, completing the PhD would never have been possible. In particular, I would like to thank my supervisor, Jochen Michaelis, and my parents, Gerd and Linda. I am incredibly grateful to have won you, Jochen, as a mentor and also as a friend. If you had known my favorite football club earlier, I probably would never have had the opportunity to write this preface. But sometimes, you need a little bit of luck. I always enjoyed the discussions with you, which went far beyond academic topics. During my time at your chair, I was able to develop academically and personally. I greatly admire how generously you deal with other people. Next to Jochen, I would especially like to thank my parents, Gerd and Linda. Not only have you supported me from the beginning wherever possible, you also always believed in me. With your caring and boundless support, I have been able to pursue my academic and sporting goals in the best possible way.

In addition, I am grateful to meet my dear colleagues, the memories with you will always be present: the DLT seminar with Andreas, countless macroeconomic lectures with Jan, the Siegen trip with Luzie, several table tennis matches with Philipp and the Crete trip with Simon and Tasso. Next to my colleagues, the unique working atmosphere at the university is also thanks to you, Heike, Rainer, Reinhold and Stefan. Furthermore, I would like to thank the participants at the 26th International Conference on Macroeconomic Analysis and International Finance on Crete, the 2nd International Conference on Payments and Settlement in Eltville and the participants at further conferences in Kassel, Rauischholzhausen and Siegen. Last but not least, I would like to thank the University of Kassel for the graduate scholarship.

1 Introduction

After the Global Financial Crisis, a number of people mistrusted the financial system and started searching for payment systems which did not require an intermediary. At the same time, someone using the pseudonym Satoshi Nakamoto (2008) created the Bitcoin. Since Bitcoin transactions are confirmed by the network, there is no need for an intermediary. Despite the new technology, the group of Bitcoin users was initially limited to insiders who mainly used the coin for trading. However, demand and attention has changed over the last decade. The market capitalization of Bitcoin increased by more than hundredfold between 2016 and 2021, peaking at over one trillion US dollar. Similar developments can be observed in further cryptocurrencies, e.g., Ripple or Ethereum. The bypassed intermediaries, central and retail banks, may thus observe a decline in their payment systems, i.e., cash and deposits. As a countermeasure, central banks are contemplating issuing their own digital currency, see Sveriges Riksbank (2021), while retail banks are seeking to improve their payment structure, see Bech and Hancock (2020) or Blocher et al. (2017).

This development raises numerous questions: do private digital payment systems have an economic impact? For instance, do they affect the value of cash so that prices in transactions with cash increase? Are they even able to replace legal currencies like cash? Under what circumstances are they able to increase welfare? On the other hand, are central banks able to tilt the playing field until they win? Should they emit their own digital currency? And if so, should the central bank digital currency (CBDC) be interest-bearing? Does an interest-bearing CBDC complement or substitute cash? Are retail banks and miners able to match the conditions for an interest-bearing CBDC to stay alive? Is a CBDC able to increase consumption and welfare?

Monetary search models can be used to tackle some of these questions. Different from classical models, e.g., cash-in-advance or money-in-utility-function models, money has an explicit function in monetary search models: namely, to simplify trade by reducing trade frictions. Despite the numerous studies which extend monetary search models by a secondary currency, the currency competition and the welfare effects are less well explored. This work aims to fill this gap.

A distinction can be made between three generations of monetary search models. In generation one, i.e., Kiyotaki and Wright (1993), goods and money are indivisible, the barter ratio is always one by one. Trejos and Wright (1995) expand generation one by divisible goods. In generation two, a Nash bargaining process decides over the quantity of goods for one monetary unit. Finally, goods as well as money are divisible in generation three, see Lagos and Wright (2005). Generation one is well suited to distinguish between partially and fully accepted currencies, e.g., cryptocurrencies and cash. Generation two is a good fit for investigating the effects of two currencies on another. For instance, to answer whether digital payment systems affect the value of cash. Finally, generation three is suitable to model further groups such as the central bank, retail banks, miners or entrepreneurs. Since all three generations are similar in some parts, this work is not free of redundancies.

Section 2 of this work focuses on two questions: firstly, under which circumstances is a secondary currency also fully accepted next to cash? And secondly, does a secondary currency increase welfare? To distinguish between partially and fully accepted currencies, the framework of Kiyotaki and Wright (1993) is used. Kiyotaki and Wright themselves address the equilibrium with two fully accepted currencies, but they do not discuss it in full. Nor does a welfare comparison with two currencies take place.

In Section 2, Jochen Michaelis and I show that results depend on two points: firstly, the properties of the new currency, i.e., degree of acceptance and monetary benefit, and secondly, the share of digital money traders replacing sellers. If the new currency is partially accepted, a welfare improvement requires that the monetary benefit of cash and the fraction of digital money traders replacing sellers are large. Since there are less sellers now, the holding duration for cash increases and a high monetary benefit for holding cash turns out to be a necessary condition. In addition, if digital money traders replace only cash holders, a welfare improvement is impossible since a fully accepted currency is exchanged for a partially accepted one. If the new currency is also fully accepted, on the other hand, both monetary benefits and the share of digital money traders replacing sellers have to be within a certain range. Otherwise, either one currency is not fully accepted or there is no welfare improvement since the share of sellers and thus the trade probability is too low.

Since money traders always get one unit of their preferred good for one monetary unit in Kiyotaki and Wright (1993), prices are constant even if money supply increases. To address the question of whether a new digital currency, e.g., a CBDC, affects prices in transactions with cash by increasing money supply further, the model of Trejos and Wright (1995) is extended by a secondary currency in Section 3. Since the traded quantity is the reciprocal of the price in generation two, a low amount of goods implies a high price and thus a currency with a low value. Thus, if the traded quantity in transactions with cash decreases with the emission of a CBDC, a CBDC affects the price level in transactions with cash in a negative way. Of course, there are already studies with two currencies building on Trejos and Wright, e.g., Camera et al. (2004) or Craig and Waller (2000), but these studies make use of a take-it-or-leave-it offer. As a consequence, the traded quantity is always the same and questions about prices cannot be answered.

In Section 3, I show that a CBDC reinforces inefficiencies in transactions with cash, in particular if a CBDC is interest-bearing. On the one hand, money supply increases further so that prices do too. On the other hand, due to the interest payment for a CBDC, opportunity costs for cash holders increase. All in all, the value of cash decreases and cash holders get a lower quantity of goods for their monetary unit. As a consequence, prices in transactions with cash increase. Nevertheless, a CBDC is able to increase welfare if there is a liquidity shortage in the single currency regime. In this case, the amount of trades and thus consumption increases.

Generation one as well as two focus on two groups of agents: sellers and buyers. To model further groups, e.g., a central bank, retail banks or miners, generation three is used in Section 4. By doing so, the model of Lagos and Wright (2005) is linked with the model of Chiu et al. (2021) and Fernández-Villaverde and Sanches (2019). Even if all studies provide numerous insights, Lagos and Wright use only cash, Chiu et al. address only deposits and a CBDC, while Fernández-Villaverde and Sanches deal only with cryptocurrencies. To discuss the currency competition, a combined framework with all four payment systems is necessary.

Combining these models and assuming that private agents use only the most economical payment systems, I show that a central bank is able to tilt the playing field until it wins by providing an interest-bearing CBDC. In this case, retail banks and miners, which provide deposits and cryptocurrencies, are forced to match the conditions for a CBDC to avoid runs. Retail banks face a lower profit but survive. Miners, on the other hand, go broke since they are not able to offer such conditions. Since retail banks offer better conditions for deposits, consumption and welfare increase.

2 Is a Secondary Currency Essential? - On the Welfare Effects of a New Currency[1]

2.1 Introduction

The process of digitalization accelerates the emergence of new currencies such as cryptocurrencies, corporate currencies and central bank digital currencies. These currencies may serve as additional medium of exchange, they are new competitors on the markets for liquidity services. Kiyotaki and Wright (1993) have shown that fiat money is essential, i.e., different from a barter economy, fiat money allows for a better resource allocation. Now, we put forward a similar question: is the secondary currency essential too? Does the introduction of a new currency allow for a welfare improvement even if a fully accepted currency is in circulation? To tackle this question, we use the dual currency search framework of Kiyotaki and Wright (1993). The answer we find is a limited yes. Not surprisingly, the scope for a welfare improvement depends on differences in returns and costs. But in addition, the sign of the welfare effect very much depends on the fraction of cash traders who will be replaced by digital money traders, or, equivalently, the degree of substitution between the new digital currency and the traditional currency.

The focus of our model is an advanced economy with a well-functioning payment system. We have in mind the Eurozone and/or the United States, where cash is an established medium of exchange and where now a cryptocurrency such as Bitcoin emerges. Another example is Switzerland, where the Euro is accepted in most parts of the country despite the universal acceptance of the Swiss franc. We do not believe in a cashless society, our framework thus assumes that cash as traditional currency

[1]This section is based on: Fuchs, Max and Jochen Michaelis (2022): Is a secondary currency essential? - On the welfare effects of a new currency. MAGKS Discussion Paper No. 5/22.

remains in circulation even if the new currency is fully accepted. We do not model the process of currency substitution with the use of the new currency instead of cash. Such a full crowding out of the domestic currency is more relevant for high-inflation countries and countries with eroding economic and political institutions. The use of multiple currencies during turbulent times, studied and surveyed in, e.g., Airaudo (2014), Calvo and Végh (1992), Giovannini and Turtelboom (1994) and Selçuk (2003), is no equilibrium phenomenon, so that the Kiyotaki-Wright framework is not appropriate. Note, however, the different view of Colacelli and Blackburn (2009), who employ the dual currency approach to investigate the multiple currency usage during the Great Depression in the United States and the 2002 recession in Argentina.

The coexistence of both cash and the secondary currency has to be an equilibrium outcome. The Kiyotaki-Wright framework shows this desirable feature. Moreover, this framework allows for the distinction between partial and full acceptance of the secondary currency. For different modeling approaches, we refer to the overlapping generation model of Lippi (2021), the currency competition model of Schilling and Uhlig (2019b) and the New Keynesian framework of Uhlig and Xie (2020).

The economics of dual currency regimes is the topic of a wide body of theoretical and empirical literature. An excellent overview of the search-theoretic foundations of the use of multiple currencies is presented by Craig and Waller (2000). Aiyagari et al. (1996) study the coexistence of money and interest-bearing securities, Camera et al. (2004) distinguish between safe and risky fiat monies, Curtis and Waller (2000) focus on the simultaneous use of legal and illegal currencies, while Lotz (2004) addresses the question how to regulate a new currency. Ding and Puzello (2020) use laboratory experiments to explore how governmental interventions such as legal restrictions on the use of a foreign currency or a change in using costs affect the circulation of the domestic currency. Also using a laboratory experimental design, Rietz (2019) analyzes the determinants of the acceptance of a secondary currency. Surprisingly, all these studies say very little about the scope of a welfare-improvement of a secondary currency. This section aims to fill this gap. The remainder of Section 2 is organized as follows. Section 2.2 describes the model setup of our analysis. Section 2.3 presents the single currency regime as benchmark economy. Section 2.4 discusses two switching scenarios, we distinguish between partial and full acceptance of the new currency. Section 2.5 concludes.

2.2 Framework

Our setup very much borrows from the dual currency framework of Kiyotaki and Wright (1993). Referring to yield differences and differences in the liquidity value, Kiyotaki and Wright (1993) show that there exist equilibria with both currencies in circulation. However, they do not discuss the transition from a single currency to a dual currency regime. But neglecting the impact of the new currency on the supply of the traditional currency turns out to be decisive for the welfare effect of a new currency. We thus modify the Kiyotaki-Wright framework in two ways, firstly, we take into account the interaction between the traditional and the new currency, and secondly, we use an economy with a fully accepted currency as initial equilibrium (benchmark).

The economy consists of a continuum of infinitely lived agents with population size normalized to unity. We follow Matsuyama et al. (1993) and assume that agents of type $i \in \{1, ..., I\}$ with $I \geq 3$, consume only goods of type i, but produce goods of type $i + 1$ (modulo I). As a consequence, there is no double coincidence of wants and no pure barter in the economy. Money is necessary for trading.

In accordance with Kiyotaki and Wright (1993) and, again, Matsuyama et al. (1993), we assume that goods production requires a consumption good as input, agents cannot produce until they have consumed. An agent produces one unit of output according to a Poisson process with constant arrival rate, α, where α measures output per unit of time. We will focus on the limiting case, $\alpha \to \infty$, so that production is instantaneous. The fraction of the population who is producer degenerates to zero, all agents are traders (see Appendix A for the dynamic structure of the model).

In addition to the commodities, the economy is endowed with two types of money, cash and digital money. We distinguish between three trading states: agents are cash traders C, digital money traders D or commodity traders (sellers S). We use the following notation. In a dual currency regime, let μ_C and μ_D be the fraction of agents endowed with one unit of cash and digital money. The fraction of commodity traders, μ_S, then is $\mu_S = 1 - \mu_C - \mu_D$. In a single currency regime, there is no digital money, $\mu_D = 0$, the fraction of cash traders is μ_C^s, the fraction of commodity traders is $\mu_S^s = 1 - \mu_C^s$. The superscript s stands for single currency regime.

Meetings are pairwise and occur according to a Poisson process with constant arrival rate, β, with $\frac{\beta}{I} = 1$.[2] Let V_j, $j = S, C, D$, be the value functions of a commodity trader, a cash trader and a digital money trader, and let $r > 0$ denote the agent's rate of time preference. The expected returns to search are then given by the Bellman Equations

[2] The assumption of a uniform random matching process where the matching of any pair of agents is equally likely, is common in the literature. Due to the randomness of meetings, agents cannot commit to a long-term agreement, credit arrangements cannot be enforced. Corbae et al. (2003) relax the assumption of random meetings, they develop a model of monetary exchange with directed search. Matsuyama et al. (1993) stick to the assumption of random meetings, but they assume a non-uniform matching process. In a two-country, two-currency model, agents are randomly paired, but the probability of meeting a domestic agent with the domestic currency exceeds the probability of meeting a foreign agent with the foreign currency. Most interesting is the case of (choice of the currency in) international pairings. The authors discuss the conditions under which either both currencies circulate or an international currency emerges.

$$rV_S = \mu_C \max_{\pi_C(i)}[\pi_C(i)(V_C - V_S)] + \mu_D \max_{\pi_D(i)}[\pi_D(i)(V_D - V_S)] \tag{2.1}$$

$$rV_C = \gamma_C + \mu_S \Pi_C(U - \eta_C + V_S - V_C) \tag{2.2}$$

$$rV_D = \gamma_D + \mu_S \Pi_D(U - \eta_D + V_S - V_D). \tag{2.3}$$

The flow return to a seller is the sum of two terms. The first term is the probability of meeting a cash trader, μ_C, times the probability of accepting cash, $\pi_C(i)$, times the gain of accepting cash, $V_C - V_S$. Note that $\pi_C(i)$ is chosen optimally by agent i. The second term is the probability of meeting a digital money trader, μ_D, times the optimally chosen probability of accepting digital money, $\pi_D(i)$, times the gain of accepting digital money, $V_D - V_S$.

If the return of switching the state is positive (negative), seller i always accepts (rejects) the currencies and sets the optimal response, $\pi_C(i)$ respective $\pi_D(i)$, to unity (zero). If sellers are indifferent between states, they flip a coin with $0 < \pi_C(i), \pi_D(i) < 1$, a currency is partially accepted.[3][4] Since, by assumption, there is no pure barter and no consumption of the own production, a positive flow return to a seller requires a switch of status from a commodity to a money trader.

[3]There are some alternatives to model partial acceptance of a currency. For instance, assume two types of sellers, A and B. Seller A always accepts a currency, while seller B always rejects the currency. The overall acceptance rate depends on the distribution across the two types. This approach may be seen as more intuitive, but needs the assumption of a fourth trading state. Based on some own calculations, we conclude that the additional insights do not warrant the additional algebra, we give precedence to simplicity.

[4]Neither the Trejos and Wright (1995) nor the Lagos and Wright (2005) framework allows for the modeling of a partially accepted currency. In the model of Trejos and Wright (1995), a buyer and a seller bargain over the quantity of goods the buyer gets for one unit of money. Partial acceptance of a currency requires that the seller is indifferent between trading and non-trading. In the bargain, the buyer can always ensure that the seller is not indifferent by offering to take an infinitesimal smaller amount of goods, or equivalently, by offering an infinitesimal higher price, see also Craig and Waller (2000). The seller always accepts, a trade always occurs, but this is the scenario of a fully accepted currency. The same line of reasoning holds for the Lagos and Wright (2005) framework.

For a cash trader, the expected return from trading is equal to the probability of meeting a seller, μ_S, times the overall acceptance of cash, Π_C, times the gain of consumption and switching status from C to S, $U - \eta_C + V_S - V_C$. Here, U denotes utility of consuming and η_C costs of using cash. If no trading takes place, the cash trader receives a permanent monetary benefit, γ_C. In the case of storage costs and/or inflation, we have $\gamma_C < 0$. For a digital money trader, the line of argument is very much the same, see Equation (2.3). Note that we name the secondary currency as digital money, but we do not model any specific feature of digital currencies. Transaction fees, a high rate of return or degree of volatility, a more speedy settlement of payments etc. are subsumed under γ_D and η_D. Moreover, a trade between a cash and a digital money trader does not make both agents better off. In case of such a meeting, both agents continue with their own money. Thus, we rule out side-payments, see also Aiyagari et al. (1996).

Our focus will be on symmetric equilibria with $\pi_C(i) = \Pi_C$ and $\pi_D(i) = \Pi_D$. In accordance with Kiyotaki and Wright (1993), welfare is defined by the expected utility of all agents before the initial endowment of money and commodities is randomly distributed among them. In terms of expected flow returns, the welfare criterion can be expressed as (see Appendix A):

$$rW = \mu_S rV_S + \mu_C rV_C + \mu_D rV_D. \tag{2.4}$$

2.3 Single Currency Regime

Despite it is a truism that the welfare effect of a new currency very much depends on the starting point (or initial equilibrium), the literature has neglected this issue. Since we are primarily interested in developed economies with a well-functioning payment system, our starting point will be a single currency regime, only cash is in circulation, and cash is fully accepted. In the initial equilibrium, there is no digital money, $\mu_D = 0$. Full acceptance of cash, $\pi_C(i) = \Pi_C = 1$, requires that the gain of accepting cash and switching the state from S to C must be positive, $V^s_C - V^s_S > 0$. For the single currency regime the Bellman Equations simplify to

$$rV^s_S = \mu^s_C(V^s_C - V^s_S) \tag{2.5}$$

$$rV^s_C = \gamma_C + \mu^s_S(U - \eta_C + V^s_S - V^s_C) \tag{2.6}$$

By combining these equations it is easy to show that the condition $V^s_C - V^s_S > 0$ is equivalent to

$$\rho^s_C \equiv \gamma_C + \mu^s_S(U - \eta_C) > 0. \tag{2.7}$$

Here, ρ^s_C is the expected per period return of cash. If the sum of the expected net utility from buying and consuming a good minus the storage costs (or plus the monetary benefit) is positive, cash will be universally accepted. Inserting Equations (2.5) and (2.6) into Equation (2.4), and observing $\mu_D = 0$, delivers the level of welfare in the single currency regime:

$$rW^s = \mu^s_C \rho^s_C. \tag{2.8}$$

Notice that the welfare effects of switching the status add up to zero. Sellers improve their welfare by switching the status from S to C, but the cash traders face an equal-sized expected loss of switching from C to S.

An increase in the money supply, in our model captured by an increase in the fraction of cash traders, has two (well-known) effects on welfare. A higher μ^s_C facilitates trade, sellers find a trading partner more easily (liquidity effect). But a higher μ^s_C means a lower μ^s_S, the number of commodities (sellers) declines. The welfare-maximizing fraction of cash traders, $(\mu_C^s)^*$, balances these effects. Observing Equation (2.7) as well as $\mu_S^s = 1 - \mu_C^s$, the derivation of Equation (2.8) with respect to μ_C^s yields

$$(\mu_C^s)^* = \frac{1}{2} + \frac{\gamma_C}{2(U - \eta_C)}. \tag{2.9}$$

Equation (2.9) extends Kiyotaki and Wright (1993), who focus on the special case $\gamma_C = 0$ with $(\mu_C^s)^* = 1/2$. Depending on the sign of γ_C (monetary benefit versus storage costs), $(\mu_C^s)^*$ exceeds or falls short of 1/2.

2.4 Two Switching Scenarios

Besides the initial equilibrium, the welfare effect also depends on the acceptance of the new currency. We distinguish between two scenarios. Firstly, cash is fully accepted and the digital currency is partially accepted (Section 2.4.1), and secondly, both currencies are fully accepted (Section 2.4.2).[5]

2.4.1 Cash Fully Accepted, Digital Money Partially Accepted

The introduction of a new currency means that digital money is part of the initial endowment, $\mu_D > 0$. As mentioned above, partial acceptance of digital money requires that sellers are indifferent between state S and state D, $V_S = V_D$. Sellers flip a coin with $0 < \pi_D(i) = \Pi_D < 1$. Denoting partial acceptance of digital money with the superscript p, the Bellman Equations are now:

[5] We do not model the way to becoming a cashless society. Cash will maintain the status of legal tender, and, even more important, central banks will not be powerless witnesses of the decline in the demand for their product. We agree with Rogoff (2017): "..., it is hard to see what would stop central banks from creating their own digital currencies and using regulation to tilt the playing field until they win. The long history of currency tells us that what the private sector innovates, the state eventually regulates and appropriates." An interesting case study is Sweden, where the usage of cash dramatically declined. But the decline is not the result of cryptocurrencies or corporate currencies, but primarily the result of the app "Swish", which allows for payments avoiding the central bank clearing system, see Sveriges Riksbank (2021).

$$rV_S^p = \mu_C^p(V_C^p - V_S^p) \tag{2.10}$$

$$rV_C^p = \gamma_C + \mu_S^p(U - \eta_C + V_S^p - V_C^p) \tag{2.11}$$

$$rV_D^p = \gamma_D + \mu_S^p \Pi_D(U - \eta_D). \tag{2.12}$$

Any comparative statics analysis needs a hypothesis on the replacement of sellers and cash traders by the digital money traders. This is done by

$$\mu_S^p = \mu_S^s - \lambda\mu_D \tag{2.13}$$

$$\mu_C^p = \mu_C^s - (1-\lambda)\mu_D, \tag{2.14}$$

where $\lambda \in [0,1]$ denotes the replacement parameter. For $\lambda = 0$, digital money traders do not replace any seller, the economy's endowment with goods remains the same, the digital money traders replace one-to-one cash traders. The new currency does not change the endowment of the economy with money, but the money supply is now made up of two fiat currencies.

For $\lambda = 1$, digital money traders replace only sellers. Since the proportion of cash traders remains constant, the new currency implies an increase in the economy's money supply. The replacement parameter serves as a measure of the degree of substitution between digital money and cash. For low values ($\lambda < 0.5$), digital money and cash are close substitutes, whereas for large values ($\lambda > 0.5$), these currencies are bad substitutes. The equilibrium acceptance rate turns out to be

$$\Pi_D = \frac{\mu_C^p \nu_C^p \rho_C^p - \gamma_D}{\mu_S^p(U - \eta_D)} \tag{2.15}$$

with $\nu_C^p \equiv 1/(1+r-\mu_D)$ and $\rho_C^p \equiv \gamma_C + \mu_S^p(U-\eta_C) > 0$. Note that Π_D is decreasing in the monetary benefit of digital money, γ_D, and increasing in the using costs, η_D. If, for instance, the monetary benefit goes up, digital money will become more attractive, the expected flow return of digital money increases and exceeds the flow return to a seller. To restore indifference between being a seller and a digital money trader requires a lower acceptance rate for digital money. In a similar vein, when the expected per period return of cash, ρ_C^p, increases, the seller's gain of switching from S to C increases, V_S^p exceeds V_D^p. Again, to restore indifference, the equilibrium acceptance rate must be higher.

Let us consider welfare. We use the Bellman Equations (2.10) to (2.12) to compute the new expected returns to search and insert the results into Equation (2.4). We yield

$$rW^p = (1 + \mu_D \nu_C^p)\mu_C^p \rho_C^p. \tag{2.16}$$

The comparison of Equation (2.16) with (2.8) starts with the polar case, $\lambda = 0$, digital money traders replace only cash traders. Then we can show that $rW^p - rW^s > 0$ requires $0 > r + \mu_S^p$. This condition is never fulfilled. Therefore, for $\lambda = 0$, the introduction of a new partially accepted currency unambiguously lowers welfare. The cash traders, who are replaced by digital money traders, switch from a currency with full acceptance to a currency with partial acceptance. The aggregate money supply does not change, but the probability of a successful match and thus the liquidity value declines. For $\lambda = 1$, where digital money traders replace only sellers, we get

$$rW^p - rW^s > 0 \quad \Rightarrow \quad \frac{\gamma_C}{r + \mu_C^s} > U - \eta_C. \tag{2.17}$$

We distinguish between three effects on welfare. Firstly, the economy is less well endowed with goods. Secondly, exchange is made easier by the increase in the money supply (liquidity). And thirdly, from the cash traders point of view, the number of trades declines, so that the expected holding period of cash goes up. For $\gamma_C \neq 0$, this matters for welfare.

For $\gamma_C = 0$, Condition (2.17) is not fulfilled, the new currency lowers welfare. Since there is a fully accepted currency already in place, the liquidity effect is positive but small. The negative endowment effect unambiguously dominates. If cash has some storage costs, $\gamma_C < 0$, the prolongation of the holding period amplifies the decline in welfare. A monetary benefit of the traditional currency and thus a positive prolongation effect, $\gamma_C > 0$, turns out to be a necessary condition for a positive welfare effect of the new currency. Note that the prolongation effect declines in both the discount rate, r, and the share of cash traders, μ_C^s. The higher μ_C^s, the longer the holding period in the initial equilibrium, and the lower is the marginal welfare effect.

The welfare-maximizing fraction of cash traders is also affected by the introduction of a new partially accepted currency. Maximizing Equation (2.16) with respect to μ_C^p yields

$$(\mu_C^p)^* = \frac{1-\mu_D}{2} + \frac{\gamma_C}{2(U-\eta_C)} = (\mu_C^s)^* - \frac{\mu_D}{2}. \tag{2.18}$$

The optimal fraction of cash traders is decreasing in the fraction of digital money traders. The optimal response to an increase in liquidity supplied by digital money traders is a decline in liquidity supplied by the cash traders. Note that this result does not depend on the replacement parameter, λ, and thus on the question whether digital money and cash are good or bad substitutes.

The replacement parameter comes into play, if the optimal response to the new currency, given by Equation (2.18), differs from the actual response assumed in Equation (2.14). The optimal response to the introduction of digital money is a decline of μ_C^p by $0.5\mu_D$, the (assumed) actual response of μ_C^p is a decline by $(1-\lambda)\mu_D$. If digital money and cash are close substitutes ($\lambda < 0.5$), the actual decline exceeds the optimal decline, and to close the gap, it is optimal to increase the cash money supply. If digital money and cash are bad substitutes ($\lambda > 0.5$), on the other hand, the actual decline of μ_C^p falls short of the optimal decline, and now it is optimal to lower the cash money supply. Proposition 2.1 summarizes.

Proposition 2.1 *Suppose that cash is fully accepted and the new currency is partially accepted. (i) If digital money and cash are very close substitutes ($\lambda \to 0$), the new currency lowers welfare. (ii) If digital money and cash are very bad substitutes ($\lambda \to 1$), a positive welfare effect requires a "strong" monetary benefit of cash. (iii) A new currency lowers the welfare-maximizing supply of cash. (iv) If the new currency primarily replaces cash (goods), the welfare-maximizing response to the new currency is an increase (a decline) in the cash money supply.*

2.4.2 Both Currencies Fully Accepted

Our second switching scenario assumes $\Pi_C = \Pi_D = 1$. Full acceptance of cash requires $V_C > V_S$, full acceptance of the digital money requires $V_D > V_S$. Rearranging the Bellman Equations (2.1) to (2.3) shows that these constraints are fulfilled if and only if

$$V_C^f > V_S^f \quad \Rightarrow \quad \rho_C^f > \mu_D \nu_D^f \rho_D^f \tag{2.19}$$

$$V_D^f > V_S^f \quad \Rightarrow \quad \rho_D^f > \mu_C^f \nu_C^f \rho_C^f \tag{2.20}$$

hold. Here, $\rho_D^f \equiv \gamma_D + \mu_S^f(U - \eta_D)$ is the expected per period return of the digital currency, and $\nu_D^f \equiv 1/(r + \mu_S^f + \mu_D)$. The superscript f denotes the dual currency regime with full acceptance of the new currency. If the expected per period return of cash does not exceed Threshold (2.19), cash will no longer be fully accepted. Similarly, if the expected per period return of the digital money does not exceed Threshold (2.20), the digital money will not be fully accepted. To put it different, the existence of an equilibrium requires that

$$\mu_C^f \nu_C^f - 1 < \frac{\rho_D^f - \rho_C^f}{\rho_C^f} < \frac{1}{\mu_D \nu_D^f} - 1 \tag{2.21}$$

holds. The relative spread between ρ_D^f and ρ_C^f must not be too big, otherwise either the digital currency or cash is no longer fully accepted. Kiyotaki and Wright (1993, p. 75) report a similar result, but the authors do not specify the interval.

Welfare in the regime of two fully accepted currencies can be computed as

$$rW^f = \mu_C^f \rho_C^f + \mu_D \rho_D^f. \tag{2.22}$$

To sign the net welfare effect of the introduction of a universally accepted new currency, we have to compare Equation (2.22) with (2.8). Again, we need a hypothesis on the replacement of sellers and cash traders by the digital money traders. We adapt Equations (2.13) and (2.14) by assuming $\mu_S^f = \mu_S^s - \lambda\mu_D$ and $\mu_C^f = \mu_C^s - (1-\lambda)\mu_D$. The condition for a positive net welfare effect is

$$\begin{gathered} rW^f - rW^s > 0 \quad \Rightarrow \quad -\Gamma_1\lambda^2 + \Gamma_2\lambda + \Gamma_3 > 0 \\ \text{with} \quad \Gamma_1 \equiv \mu_D(U - \eta_C), \quad \Gamma_2 \equiv \gamma_C + (\mu_D + \mu_S^s - \mu_C^s)(U - \eta_C) - \mu_D(U - \eta_D) \\ \text{and} \quad \Gamma_3 \equiv \gamma_D + \mu_S^s(U - \eta_D) - [\gamma_C + \mu_S^s(U - \eta_C)]. \end{gathered} \tag{2.23}$$

Suppose digital money and cash are very close substitutes, so that digital money traders replace only cash traders, whereas the number of sellers remains constant, $\lambda = 0$. In this case, Condition (2.23) boils down to $\Gamma_3 > 0$. The cash traders, who switch status from C to D, switch to a currency with the same liquidity value (acceptance rate), they gain $\gamma_D + \mu_S^s(U - \eta_D)$, they lose $\gamma_C + \mu_S^s(U - \eta_C)$. If the former exceeds the latter, the economy yields a payoff. If digital money and cash are bad substitutes, digital money traders replace only sellers, $\lambda = 1$. Condition (2.23) simplifies to $\rho_D^f > \mu_S^s(U - \eta_C)$. The sellers, who switch status from S to D, gain ρ_D^f. But the cash traders face a loss. Since there is a lower number of sellers, the probability of exchange and consumption declines.

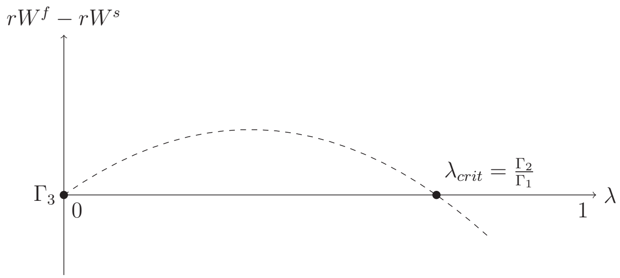

Figure 2.1: Solution to $rW^f - rW^s > 0$

Net welfare is a quadratic function in λ. Depending on λ, the sign of the net welfare effect may change. Figure 2.1 illustrates this, we assume $\Gamma_3 = 0$. For $\lambda = 0$, the new currency is neutral with respect to welfare. As λ increases, so does the sum of cash and digital money (aggregate money supply). Therefore, an increase in λ very much resembles an increase in money supply in the Kiyotaki-Wright (1993) framework. Endowing more agents with money facilitates exchange and improves welfare, the net welfare effect becomes positive. But endowing more agents with money is equivalent to endowing fewer agents with commodities, consumption and welfare go down. If the replacement parameter, λ, exceeds a critical value, $\lambda_{crit} = \Gamma_2/\Gamma_1$, the net welfare effect switches the sign and turns into negative.

Two remarks are in order: firstly, the higher the fraction of cash traders in the initial equilibrium, μ_C^s, the lower is the welfare-enhancing liquidity effect of a new currency, and the more important is the negative effect of the lower number of commodities, λ_{crit} declines, the probability of a negative net welfare goes up. Secondly, λ_{crit} may be larger than one. In this case, we observe a net welfare gain for all $\lambda \in (0, 1]$. The welfare effects of a relaxation of the assumption $\Gamma_3 = 0$ are straightforward, in Figure 2.1 the net welfare curve shifts up ($\Gamma_3 > 0$) or down ($\Gamma_3 < 0$). Since there are no novel and crucial insights, we skip the discussion.

In a world with two fully accepted currencies, the welfare-maximizing fraction of cash traders is given by:

$$(\mu_C^f)^* = \frac{1-\mu_D}{2} + \frac{\gamma_C}{2(U-\eta_C)} - \frac{\mu_D(U-\eta_D)}{2(U-\eta_C)} = (\mu_C^p)^* - \frac{\mu_D(U-\eta_D)}{2(U-\eta_C)}. \tag{2.24}$$

As shown above, the optimal response to the introduction of a partially accepted currency is a decline in the supply of cash (fraction of cash traders) by $0.5\mu_D$. If instead the new currency is fully accepted, its liquidity value is even higher, so that the decline in the optimal supply of cash is even stronger. We get

Proposition 2.2 *Suppose that both cash and the new currency are fully accepted. (i) The existence of an equilibrium requires that the relative spread between ρ_D^f and ρ_C^f fulfills Condition* (2.21)*. (ii) If digital money and cash are very close substitutes ($\lambda \to 0$), a positive spread ensures a net welfare gain. (iii) The lower the degree of substitution between digital money and cash (increasing λ), the higher the probabil-*

ity of a negative net welfare effect. (iv) A new fully accepted currency lowers the welfare-maximizing supply of cash more than the introduction of a partially accepted currency.

2.5 Conclusion

Digital currencies are on the rise. Our analysis provides insight into the welfare effects of this development. Using an economy with a fully accepted currency as benchmark, we identify the conditions under which the introduction of a secondary (digital) currency improves welfare. A decisive factor turns out to be the degree of substitution between the new currency and cash, this factor determines how many agents switch from an endowment with cash to an endowment with digital money. Our results may serve as a helping hand for the government to the question how to regulate a new currency. Of course, our framework is too simple to draw far-reaching policy conclusions, extensions are necessary. However, we are at the starting point of a fruitful discussion of the economic consequences of digital currencies. Two promising lines of research are the impact on financial intermediation, for an overview see Thakor (2020), and the macroeconomic consequences of a central bank digital currency, see, e.g., Barrdear and Kumhof (2021) or Fegatelli (2022).

But a secondary currency does not only affect welfare, it also affects the value of the first currency by increasing money supply or causing opportunity costs for holding the first currency. Since goods and money are indivisible in Kiyotaki and Wright (1993), the barter ratio is always one by one. As a consequence, prices and thus the value of a currency are constant even if money supply increases. Since goods are divisible in Trejos and Wright (1995), the model is used in Section 3. In this case, it can be answered whether a secondary currency also affects the value of the first currency.

Appendix A: Dynamic Structure of the Model

The dynamic structure of our model is visualized in Figure 2.2.

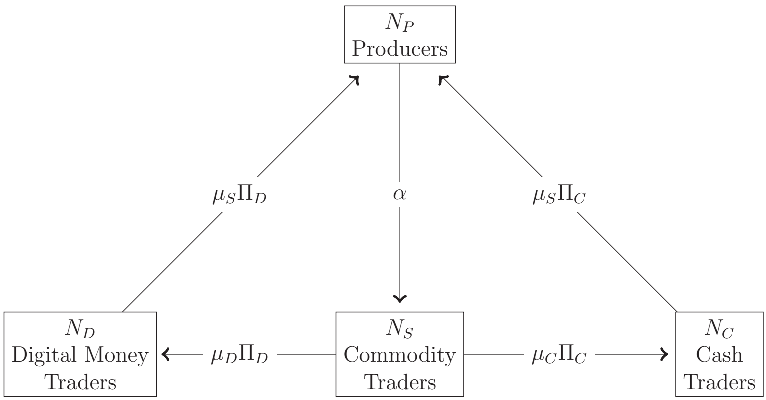

Figure 2.2: Dynamic Structure

Here, N_P, N_S, N_C and N_D denote the proportions of the population who are producers, commodity traders (sellers), cash traders and digital money traders. Producers are no traders, we thus denote μ_S, μ_C and μ_D as proportions of traders who are commodity traders (sellers), cash traders and digital money traders. A steady state (flow equilibrium) requires an equal flow out of and into a knot. For producers, cash traders and digital money traders we get:

$$\alpha N_P = \mu_S \Pi_C N_C + \mu_S \Pi_D N_D \tag{A1}$$

$$\mu_S \Pi_C N_C = \mu_C \Pi_C N_S \tag{A2}$$

$$\mu_S \Pi_D N_D = \mu_D \Pi_D N_S, \tag{A3}$$

where Π_C (Π_D) is the overall acceptance of cash (digital money). Observing $N_P + N_S + N_C + N_D = 1$ as well as $\mu_S + \mu_C + \mu_D = 1$, Equations (A1) to (A3) deliver

$$N_P = 1 - \frac{\alpha}{\alpha + \mu_S \mu_C \Pi_C + \mu_S \mu_D \Pi_D}. \tag{A4}$$

As mentioned in the text, we focus on the limiting case, $\alpha \to \infty$, so that production is instantaneous, and the equilibrium number of producers approaches zero, $N_P = 0$. It immediately follows that $N_S = \mu_S$, $N_C = \mu_C$ and $N_D = \mu_D$.

As also mentioned in the text, we define welfare by the expected utility of all agents before the initial endowment is randomly distributed among them:

$$W = N_P V_P + N_S V_S + N_C V_C + N_D V_D.$$

Inserting our results leads to Equation (2.4), where the welfare criterion is expressed in terms of expected flow returns.

In order to compare these fractions with their analogues in a single currency regime, we redo the analysis with $N_D = \mu_D = 0$. This delivers $N_P^s = 0$, $N_S^s = \mu_S^s$ and $N_C^s = \mu_C^s$, where the superscript s stands for single currency regime. The link between the fractions in the single and the dual currency regime (with partial acceptance of the secondary currency) is given by Equations (2.13) and (2.14).

3 Does a CBDC Reinforce Inefficiencies?[6]

3.1 Introduction

Monetary search models show that the first best allocation, the welfare-maximizing quantity of goods where marginal utility equals marginal cost, is missed due to inefficiencies. The inefficiencies either occur owing to discounting or a suboptimal amount of money. In the case of discounting, sellers produce less since costs of production are incurred today, but the utility of consumption is enjoyed tomorrow. If money supply is too high, sellers also produce less since they know that they are in the minority and thus in a strong market position. If a central bank issues a central bank digital currency (CBDC) to tilt the playing field for rising private digital payment systems, money supply and thus inefficiencies in transactions with cash could increase further. This effect would be even stronger if a CBDC is interest-bearing. To answer whether an interest-bearing CBDC reinforces inefficiencies in transactions with cash, the monetary search model of Trejos and Wright (1995, henceforth TW) is extended by a CBDC. I show that an interest-bearing CBDC reinforces inefficiencies in transactions with cash since money supply and opportunity costs for cash holders increase. Nevertheless, the gradual introduction of a CBDC improves welfare, the gains of the "new" CBDC holders exceed the loss of the remaining sellers and cash holders. This holds true for most plausible parameter constellations.

As generation two of monetary search models, the TW framework is the only one which enables an examination of the effects of two currencies on each other. In generation one, i.e., Kiyotaki and Wright (1993), goods and money are indivisible. As a consequence, the value of a currency, that is the quantity of goods a buyer

[6]This section is based on: Fuchs, Max (2022): Does a CBDC reinforce inefficiencies? MAGKS Discussion Paper No. 28/22.

receives for one monetary unit, is always one. Generation one is mainly used to distinguish between partially and fully accepted currencies. For instance, Fuchs and Michaelis (2023) show that a partially (fully) accepted currency, which circulates as a secondary currency next to fully accepted cash, increases welfare if the secondary currency and cash are complements (substitutes). In generation three, i.e., Lagos and Wright (2005), money demand is not affected by properties of other currencies. Generation three is mainly used to model further groups next to sellers and buyers, see, e.g., Chiu et al. (2021), Fernández-Villaverde and Sanches (2019) or Fuchs (2022a).

Of course, there are already studies with two currencies building on TW, e.g., Camera et al. (2004) or Craig and Waller (2000), but these studies make use of a take-it-or-leave-it offer. In this case, buyers purchase the quantity where costs of production are equal to the surplus of switching position for sellers. As a consequence, the welfare-maximizing quantity of goods is always traded and questions about inefficiencies cannot be answered.

The structure of this section is as follows: Section 3.2 deals with the framework and, in particular, with the traded quantity in transactions with cash. Section 3.3 describes the dual currency regime and examines how a CBDC affects the traded quantity in transactions with cash. Section 3.4 compares welfare between a single and a dual currency regime. Finally, Section 3.5 concludes.

3.2 Framework

As in TW, there is a [0,1]-continuum of agents which is divided into sellers $1-\mu$ and buyers $\mu \in (0,1)$. In the initial stage, sellers have no endowment while buyers have one monetary unit. Agents of type $i \in \{1, ..., I\}$ with $I \geq 3$, prefer only goods of type i but produce goods of type $i+1$ (modulo I), see also Matsuyama et al. (1993). Thus, nobody consumes own production, pure barter does not take place and money is necessary for trading. As soon as buyers meet sellers who are able to produce the preferred good of buyers, a Nash bargaining process decides about the traded quantity for one monetary unit. Afterwards, sellers start to produce and receive one monetary unit. In the next period, sellers act as buyers (and vice versa).

3.2.1 Bellman Equations

Now, buyers are looking for sellers who produce their preferred good, while sellers are looking for buyers who demand their production. Meetings are pairwise and occur according to a Poisson process with constant arrival rate, β, with $\beta/I = 1$. With $r > 0$ as discount rate, the Bellman Equations are

$$rV_s(Q) = \mu[V_c(Q) - V_s(Q) - c(q)] \tag{3.1}$$

$$rV_c(Q) = (1-\mu)[V_s(Q) - V_c(Q) + u(q)] + \gamma_c, \tag{3.2}$$

where $V_s(Q)$ and $V_c(Q)$ denote the expected return for sellers and buyers (cash holders). The subscript s (c) denotes sellers (cash holders). Note that V_s and V_c depend on Q, which is the traded quantity on macroeconomic level. The traded quantity on microeconomic level is denoted by q. Equation (3.1) displays the Bellman Equation of a seller. With a probability of μ a trade with a buyer takes place. A seller has a surplus of switching position, $V_c - V_s > 0$, minus costs, $c(q)$, of producing quantity q with $c(0) = 0$, $c'(q) > 0$ for $q > 0$, $c''(q) \geq 0$ and $c'(0) = 0$.

On the other side, a buyer trades with a probability of $1-\mu$, has a loss of switching position, $V_s - V_c < 0$, and utility, $u(q)$, of consuming quantity q with $u(0) = 0$, $u'(q) > 0$, $u''(q) < 0$ and $u'(0) > 0$, see Equation (3.2). Thus, q is the quantity a buyer receives for one monetary unit. The reciprocal is the price, $p = 1/q$. To ensure the existence of a monetary equilibrium (see below), I assume a monetary benefit for holding cash, $\gamma_c > 0$.

3.2.2 Nash Bargaining Process

As mentioned above, if buyers find adequate sellers who are able to produce the preferred good of buyers, a Nash bargaining process takes place to determine q:

$$\max_q [\underbrace{V_s(Q) - V_c(Q) + u(q)}_{\text{buyer's surplus}}]^{\theta} [\underbrace{V_c(Q) - V_s(Q) - c(q)}_{\text{seller's surplus}}]^{1-\theta}, \tag{3.3}$$

where V_c (V_s) denotes the threat point for buyers (sellers), see also TW. Here, $\theta \in (0, 1)$ is a buyer's bargaining power. For $\theta \to 1$, buyers make a take-it-or-leave-it offer and purchase the quantity where costs of production, $c(q)$, are equal to the surplus of switching position, $V_c - V_s$, for sellers. In this case, sellers make neither profits nor losses, they are indifferent between selling or not, from Equation (3.1) I get $V_s = 0$. Assuming that they are producing, the buyer's surplus now coincides with the overall trade surplus, defined by $\Delta(q) \equiv u(q) - c(q) > 0$. They buy the the welfare-maximizing quantity, q^*, which satisfies $u'(q^*) = c'(q^*)$. Note that the traded quantity will always be lower than $\overline{q}$, where $\overline{q} > 0$ is defined by $u(\overline{q}) = c(\overline{q})$.

Let us have a closer look at the maximization problem (3.3). In the bargaining process, agents take the traded quantity on macroeconomic level as given, $V_s(Q)$ and $V_c(Q)$ are exogenous. The first order condition is

$$\frac{u'(q)}{c'(q)} = \frac{1-\theta}{\theta} \frac{\hat{\psi}(q)}{\hat{\phi}(q)} = \frac{1-\theta}{\theta} \frac{u(q) - [V_c(Q) - V_s(Q)]}{[V_c(Q) - V_s(Q)] - c(q)}, \tag{3.4}$$

where $\hat{\psi}(q) \equiv u(q) - [V_c(Q) - V_s(Q)]$ is the bargaining surplus of a buyer and $\hat{\phi}(q) \equiv [V_c(Q) - V_s(Q)] - c(q)$ is the bargaining surplus of a seller. A trade takes place only if both trade surpluses are positive (participation constraints). Since buyers' utility of consumption has to exceed the loss of switching position, the traded quantity has to exceed a threshold q_{min}, where q_{min} is defined by $\hat{\psi}(q_{min}) = 0$. Similarly, since sellers' surplus of switching position has to exceed costs of production, the costs of production and hence the produced and traded quantity must be lower than a threshold q_{max}, where q_{max} is defined by $\hat{\phi}(q_{max}) = 0$.

Given the assumptions on $u(q)$ and $c(q)$, the left-hand side of Equation (3.4) is strictly decreasing in q, and the right-hand side is strictly increasing in q for all q. Thus, on microeconomic level there is a unique monetary equilibrium for all $q \in (q_{min}, q_{max})$. For $q < q_{min}$ ($q > q_{max}$), the surplus of a buyer (seller) would be negative. By withdrawing from negotiations such a loss is avoided.

On the macroeconomic level, the traded quantity Q and thus the value of switching position, $V_c(Q) - V_s(Q)$, is endogenous. In equilibrium, $q = Q$ holds. Making use of the Bellman Equations, the first order condition now reads

$$\frac{u'(Q)}{c'(Q)} = \frac{1-\theta}{\theta}\frac{\psi(Q)}{\phi(Q)} = \frac{1-\theta}{\theta}\frac{\mu\Delta(Q) + ru(Q) - \gamma_c}{(1-\mu)\Delta(Q) - rc(Q) + \gamma_c} \tag{3.5}$$

with

$$\psi(Q) = \frac{\mu\Delta(Q) + ru(Q) - \gamma_c}{1+r} \quad \text{and} \quad \phi(Q) = \frac{(1-\mu)\Delta(Q) - rc(Q) + \gamma_c}{1+r}. \tag{3.6}$$

In order to discuss the existence and the uniqueness of a monetary equilibrium on the macroeconomic level, let me rewrite the first order condition (3.5) as $T(Q) = 0$

with

$$T(Q) \equiv \theta\phi(Q)u'(Q) - (1-\theta)\psi(Q)c'(Q). \tag{3.7}$$

The function $T(Q)$ is visualized in Figure 3.1.

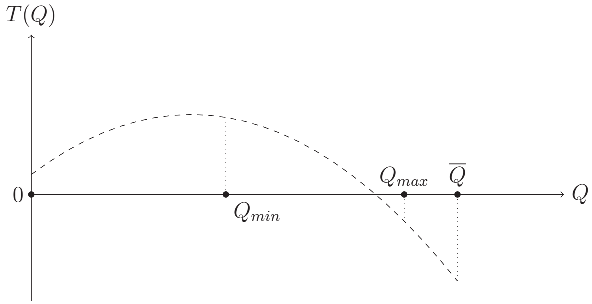

Figure 3.1: Monetary Equilibrium

For $Q = 0$, I get $T(0) = \theta\gamma_c u'(0)$, the sign corresponds to the sign of γ_c, which, by assumption, is positive. The maximum quantity is $\overline{Q}$, where $\overline{Q}$ is defined by $u(\overline{Q}) = c(\overline{Q})$. For $Q = \overline{Q}$, I get $T(\overline{Q}) = K[\gamma_c - rc(\overline{Q})]$, where K is a positive constant. If $\gamma_c = rc(\overline{Q}) = ru(\overline{Q})$, I have $\psi(\overline{Q}) = \phi(\overline{Q}) = 0$ and thus $T(\overline{Q}) = 0$. No agent gets a surplus, the equilibrium is not meaningful. For $\gamma_c > rc(\overline{Q}) = ru(\overline{Q})$, there will be no equilibrium, the buyers' participation constraint is violated, $\psi(\overline{Q}) < 0$. A very large monetary benefit means that buyers have a "golden nugget", their loss of switching position is larger than the utility of consumption even the maximum amount, $\overline{Q}$. Hence, I assume $\gamma_c < rc(\overline{Q}) = ru(\overline{Q})$, so that $T(\overline{Q}) < 0$. By continuity, there exists at least one $Q \in (0, \overline{Q})$ such that $T(\overline{Q}) = 0$, see Figure 3.1.

For a given level of γ_c with $0 < \gamma_c < rc(\overline{Q})$, there is, similar to the microeconomic level, an interval for Q that generates bargaining gains for both, buyers and sellers, $Q \in (Q_{min}, Q_{max})$. Here, Q_{min} is defined by $\psi(Q_{min}) = 0$, and Q_{max} is defined by $\phi(Q_{max}) = 0$. Observing (3.7), one can show $T(Q_{min}) = \theta[u(Q_{min}) - c(Q_{min})]u'(Q_{min}) > 0$ and $T(Q_{max}) = -(1-\theta)[u(Q_{max}) - c(Q_{max})]c'(Q_{max}) < 0$. There exists a monetary equilibrium with $Q \in (Q_{min}, Q_{max})$, see Figure 3.1.

Note that Q_{min} and Q_{max} depend on the monetary benefit, γ_c. Suppose that γ_c increases, so that the value of switching position, $V_c(Q)-V_s(Q)$, increases. The money that sellers get is more valuable, sellers are ready to produce a higher quantity. In Figure 3.1, Q_{max} shifts to the right. If γ_c goes to the upper limit, $rc(\overline{Q})$, then Q_{max} approaches to $\overline{Q}$. On the other hand, if the monetary benefit, γ_c, decreases, buyers' loss of switching position declines, so does the minimum amount of goods, Q_{min}, that compensates for this loss. In Figure 3.1, Q_{min} shifts to the left. If γ_c goes to the lower limit zero, then Q_{min} goes to zero.

The monetary equilibrium is unique, if there is only one intersection of $T(Q)$ with the abscissa in the interval between Q_{min} and Q_{max}. Unfortunately, and in contrast to the microeconomic equilibrium, the assumptions on $u(Q)$ and $c(Q)$ mentioned above are not sufficient to ensure uniqueness. Figure 3.2 illustrates. The left-hand side of Equation (3.5) is decreasing in Q for all Q. The right-hand side of Equation (3.5), however, does not need to increase in Q for $Q \in (Q_{min}, Q_{max})$. But I suppose that a positive slope is more plausible than a negative slope. A positive slope means that the ratio of the buyers' surplus to the sellers' surplus increases in Q. Or to put it simpler, buyers are interested in a higher quantity whereas sellers are interested in a lower quantity. This scenario corresponds to economic intuition. But note that a negative slope is not totally obscure. Buyers may be interested in a lower quantity, since they know that after switching position they become sellers who have to produce a large quantity. Similarly, sellers may be interested in a larger quantity, since they anticipate that they will become buyers in the future.

Figure 3.2 assumes a positive slope, i.e., the ratio $\psi(Q)/\phi(Q)$ is increasing in Q. In this case the right-hand side of Equation (3.5) intersects the left-hand side from below. If, in contrast, the ratio $\psi(Q)/\phi(Q)$ is decreasing in Q, the right-hand side intersects the left-hand side from above. From my point of view, this scenario is economically implausible, so that it will not be pursued further.

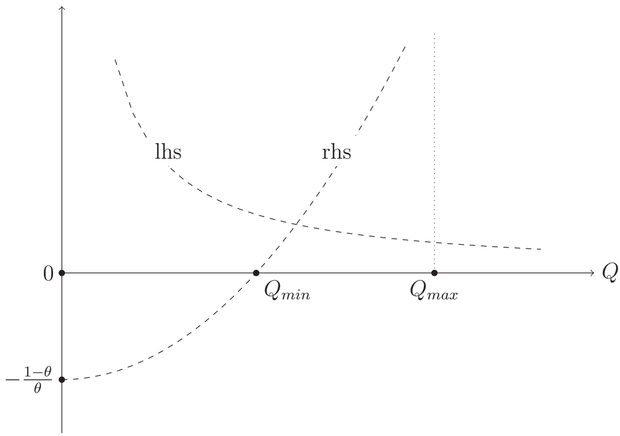

Figure 3.2: Unique Monetary Equilibrium

The welfare-maximizing quantity of goods, Q^*, is traded if marginal utility equals marginal costs so that $\rho(Q) \equiv \frac{u'(Q)}{c'(Q)} = 1$. This is true, for instance, if the bargaining power and trade surpluses of sellers and buyers are equal, $\theta = 1/2$ and $\phi = \psi$. Equation (3.5) implies several insights:

- Firstly, Equation (3.5) captures the fact that buyers (sellers) accept higher (lower) prices if their surplus increases.

- Secondly, the notation describes the positive relation between the amount of money and the price level. If μ increases, a buyer's surplus increases. Buyers know about the challenge of finding an adequate seller. If they find sellers who are able to produce the preferred good of buyers, they are willing to pay more.

- Thirdly, if the bargaining power of sellers or the discount rate increases, prices also do. If the bargaining power of sellers increases, sellers are in a better position. In addition, if the discount rate increases, sellers' earnings are discounted higher. In both cases sellers react by raising prices.

- Fourth, if the monetary benefit increases, sellers are compensated more highly for costs of production, sellers are willing to produce a higher quantity (accept a lower price).

Proposition 3.1 *The welfare-maximizing quantity, Q^*, is traded if marginal utility equals marginal costs, $\rho = 1$. For $\rho > 1$, the traded quantity is too small, $Q < Q^*$. In this case, an increasing discount rate, an increasing money supply, a decreasing monetary benefit and a decreasing buyer bargaining power reinforce inefficiencies, Q goes down. For $\rho < 1$, all factors also reduce Q but inefficiencies decrease.*

3.2.3 Welfare

If a trade takes place, Q^* is the welfare-maximizing quantity. But welfare is also affected by the number of trades. In general, welfare is given by average utility. Wealth of sellers is weighted by $1 - \mu$, while wealth of buyers is weighted by μ. Rearranging yields welfare in a single currency regime (SR)

$$rW^{SR} = \mu(1-\mu)\Delta(Q) + \mu\gamma_c. \tag{3.8}$$

Welfare is given by the trade probability times the overall trade surplus plus the share of cash holders times their monetary benefit. Now, $\rho \neq 1$ implies $\Delta(Q) < \Delta(Q^*)$. Thus, for given μ and γ_c, welfare is below the case where Q^* is traded.

In TW the bargaining power is equal, $\theta = 1/2$, while there is no monetary benefit, $\gamma_c = 0$. In this case, there is a trade-off between maximizing the number of trades and optimizing the traded quantity. TW distinguish between a liquidity and price level effect. As long as there is a liquidity shortage, $\mu < 1/2$, an increase in money supply increases the number of trades (liquidity effect) but also raises prices (price level effect). Now, the number of trades is maximized if $\mu = 1/2$ which implies a trade probability of $\mu(1-\mu) = 1/4$. But $\mu = 1/2$ also implies that the traded quantity is below the welfare-maximizing one. In this case, there are too many buyers and the goods market competitiveness is too small. As a consequence, sellers do not produce the welfare-maximizing quantity of goods. Indeed, the welfare-maximizing quantity of goods is produced only if $\mu = \frac{1}{2} - \frac{r(u+c)}{2}$. In this case, the goods market is more competitive so that sellers are willing to produce the welfare-maximizing quantity, with the disadvantage of a lower number of trades since $\mu(1-\mu) < 1/4$.

Proposition 3.2 *For $\theta = 1/2$ and $\gamma_c = 0$, there is a trade-off between maximizing the number of trades and optimizing the traded quantity. If $\mu = 1/2$, the number of trades is maximized, but the traded quantity is below the welfare-maximizing one. On the other hand, Q^* requires $\mu = \frac{1}{2} - \frac{r(u+c)}{2\Delta}$, so that the number of trades is lower than optimal.*

Suppose that γ_c may serve as a policy parameter. In this case, the number of trades as well as the traded quantity can be optimized simultaneously. By choosing $\mu = 1/2$ first, the number of trades is maximized. In a second step, the monetary benefit is determined by $\gamma_c = \frac{r(u+c)}{2}$ so that Q^* is traded. Different from the previous situation ($\gamma_c = 0$) where Q^* requires $\mu < 1/2$, the goods market is less competitive now. But sellers are still willing to produce Q^* since they are compensated for their losses due to accepting lower prices by receiving a subsidy.

3.3 Dual Currency Regime

Now, the dual currency regime (DR) with a secondary currency, i.e., a CBDC, is considered next. The continuum of agents remains but there are three types now. Next to sellers, μ_s, and cash traders, μ_c, there is a fraction of agents receiving CBDC. It is assumed that $\lambda \in (0, 1)$ of the agents from an SR receive a CBDC. Thus, the fractions of agents are given by $\mu_s = (1 - \lambda)(1 - \mu)$, $\mu_c = (1 - \lambda)\mu$ and $\mu_d = \lambda$. The subscripts s, c and d denote sellers, cash holders and CBDC (digital money) holders.

3.3.1 Bellman Equations

The Bellman Equations of a seller, cash holder and CBDC holder are

$$rV_s(Q_c, Q_d) = \mu_c[V_c(Q_c) - V_s(Q_c, Q_d) - c(q_c)] + \mu_d[V_d(Q_d) - V_s(Q_c, Q_d) - c(q_d)]$$
$$rV_c(Q_c) = \mu_s[V_s(Q_c, Q_d) - V_c(Q_c) + u(q_c)] + \gamma_c$$
$$rV_d(Q_d) = \mu_s[V_s(Q_c, Q_d) - V_d(Q_d) + u(q_d)] + \gamma_d,$$

where Q_c and Q_d (q_c and q_d) are the traded quantities on macroeconomic (microeconomic) level. Sellers have two options to sell their goods: with a probability of μ_c they meet a cash holder, with μ_d they meet a CBDC holder. Buyers, who are now divided into cash and CBDC holders, search for an adequate seller and receive a monetary benefit, γ_c respective γ_d. A trade between a cash and a CBDC holder does not make both agents better off. Since I rule out side-payments, money traders continue with their own money, see also Aiyagari et al. (1996).

Different from cash, a CBDC can be interest-bearing. Since the monetary benefit covers all properties of a currency, e.g., an interest payment, I assume that the difference in the monetary benefits between a CBDC and cash, $\delta \equiv \gamma_d - \gamma_c$, is positive, $\delta > 0$ holds. In this way, one can argue that a CBDC is a "better" currency than cash. The gap in the monetary benefits, δ, also covers opportunity costs for cash holders. As long as they hold cash, they hold a currency with a lower monetary benefit. Only after consuming and switching the status from a buyer to a seller, they are able to sell their goods for CBDC.

3.3.2 Monetary Equilibrium

Different from previous studies which deal with TW and two currencies, e.g., Camera et al. (2004) or Craig and Waller (2000), there is no take-it-or-leave-it offer by the buyers here. In this way, the model is more realistic but less tractable. On the microeconomic level, where agents take Q_c and Q_d as exogenous, the bargain between a seller and a cash (CBDC) holder determines q_c (q_d). Similar to the single currency regime, it is straightforward to show that the bargaining solutions, q_c and q_d, lie within the intervals $q_c^{min} < q_c < q_c^{max}$ and $q_d^{min} < q_d < q_d^{max}$, respectively. The upper limits stem from the seller's participation constraint. A seller produces only if the costs of production are below the surplus of switching position. Thus, the produced and traded quantity must not be too large. The lower limits follow from the participation constraints of money holders. The utility from consumption has to exceed the loss of switching position, otherwise money holders do not buy.

Let us turn to the macroeconomic equilibrium with $q_c = Q_c$ and $q_d = Q_d$. Since I have two Nash bargains, sellers vs. cash holders and sellers vs. CBDC holders, I have two first order conditions, $T(Q_c) = 0$ and $Z(Q_d) = 0$, with

$$T(Q_c) = \theta_c \phi_c(Q_c) u'(Q_c) - (1 - \theta_c) \psi_c(Q_c) c'(Q_c)$$
$$Z(Q_d) = \theta_d \phi_d(Q_d) u'(Q_d) - (1 - \theta_d) \psi_d(Q_d) c'(Q_d).$$

Rearranging the Bellman Equations delivers the bargaining surpluses in a trade with cash:

$$\begin{aligned} \phi_c(Q_c) &\equiv \frac{1}{1+r} \left[\mu_s \Delta(Q_c) - rc(Q_c) + \gamma_c - \frac{\mu_d}{r + \mu_s} (\delta + \tau) \right] \quad \text{and} \\ \psi_c(Q_c) &\equiv \frac{1}{1+r} \left[(1 - \mu_s) \Delta(Q_c) + ru(Q_c) - \gamma_c + \frac{\mu_d}{r + \mu_s} (\delta + \tau) \right] \end{aligned} \tag{3.9}$$

where δ depicts the opportunity costs of sellers from accepting cash (e.g. foregone interest payments) and $\tau \equiv \mu_s [u(Q_d) - u(Q_c)] - (r + \mu_s)[c(Q_d) - c(Q_c)] \lesseqgtr 0$ are the opportunity costs of sellers from producing and selling Q_c instead of Q_d. I assume that accepting cash means accepting a "worse" currency, i.e., the sum of opportunity costs is assumed to be positive, $\delta + \tau > 0$. Accepting cash prevents the possibility of receiving digital money in the next period. From the cash holders' point of view, a trade with sellers avoids these opportunity costs, they get rid of the "worse" currency, their bargaining surplus, ψ_c, increases in $(\delta + \tau)$. The participation constraints, $\phi_c > 0$ and $\psi_c > 0$, ensure that sellers accept cash and that cash holders trade.

In a trade with CBDC the bargaining surpluses are

$$\begin{aligned} \phi_d(Q_d) &\equiv \frac{1}{1+r} \left[\mu_s \Delta(Q_d) - rc(Q_d) + \gamma_d + \frac{\mu_c}{r + \mu_s} (\delta + \tau) \right] \quad \text{and} \\ \psi_d(Q_d) &\equiv \frac{1}{1+r} \left[(1 - \mu_s) \Delta(Q_d) + ru(Q_d) - \gamma_d - \frac{\mu_c}{r + \mu_s} (\delta + \tau) \right]. \end{aligned} \tag{3.10}$$

Here, sellers have an additional benefit, $\frac{\mu_c}{r+\mu_s}(\delta+\tau)$, by accepting and getting a "better" currency. CBDC holders, on the other hand, have an additional loss by losing the better currency. The participation constraints, $\phi_d > 0$ and $\psi_d > 0$, ensure that a trade takes place.

Regarding the existence and uniqueness of the monetary equilibrium, the line of argumentation essentially follows the single currency regime. A monetary equilibrium requires both, $T(Q_c) = 0$ and $Z(Q_d) = 0$. Let us first consider $T(Q_c) = 0$. For $Q_c = 0$, I get $T(0) = \theta_c \left[\gamma_c - \frac{\mu_d}{r+\mu_s}(\delta+\tau)\right] u'(0)$, which I assume to be positive. For the maximum quantity, $\overline{Q}_c$, I get $T(\overline{Q}_c) = K_c \left[\gamma_c - \frac{\mu_d}{r+\mu_s}(\delta+\tau) - rc(\overline{Q}_c)\right]$, where K_c is a positive constant. If the monetary benefit of cash minus the opportunity costs of cash is positive but lower than the upper limit, $rc(\overline{Q}_c)$,

$$0 < \gamma_c - \frac{\mu_d}{r+\mu_s}(\delta+\tau) < rc(\overline{Q}_c),$$

then there exists at least one $Q_c \in (0, \overline{Q}_c)$ such that $T(Q_c) = 0$.

Next consider the first order condition $Z(Q_d) = 0$. For $Q_d = 0$, I get $Z(0) = \theta_d \left[\gamma_d + \frac{\mu_c}{r+\mu_s}(\delta+\tau)\right] u'(0)$, which is clearly positive. For the maximum quantity, $\overline{Q}_d$, I get $Z(\overline{Q}_d) = K_d \left[\gamma_d + \frac{\mu_c}{r+\mu_s}(\delta+\tau) - rc(\overline{Q}_d)\right]$, where K_d is a positive constant. If the monetary benefit of digital money plus the additional benefit mentioned above is lower than the upper limit, $rc(\overline{Q}_d)$,

$$\gamma_d + \frac{\mu_c}{r+\mu_s}(\delta+\tau) < rc(\overline{Q}_d),$$

there exists at least one $Q_d \in (0, \overline{Q}_d)$ such that $Z(Q_d) = 0$. Concerning the uniqueness of the monetary equilibrium, I refer to the discussion in Section 3.2.2.

3.3.3 Price Level for Transactions with Cash

Similar to a single currency regime, the inverse money demand function, ρ_c, and the traded quantity, Q_c, in a trade with cash in the dual currency regime is given by

$$\rho_c(Q_c) = \frac{(1-\theta_c)}{\theta_c}\frac{\psi_c(Q_c)}{\phi_c(Q_c)}.$$

Here, $\mu_d = 0$ implies $\rho(Q) = \rho_c(Q_c)$. The comparative statics of the monetary equilibrium is analogous to Section 3.2.2. If the money supply, $1-\mu_s$, or the gap in the monetary benefits, δ, increases, the price level in transactions with cash does too since sellers value cash less. This effect is enhanced if the seller's bargaining power, $1-\theta_c$, increases since sellers are able to raise prices further in this case. Thus, for $\rho_c > 1$, an increasing money supply, a rising gap in the monetary benefits and an increasing seller's bargaining power reinforce inefficiencies. For $\rho_c < 1$, all factors have the same effect on Q_c but inefficiencies decrease since $Q_c > Q_c^*$.

Compared to an SR, buyers have a higher surplus since they are able to pay with a "worse" payment system, $\psi_c(Q_c)$ increases. By paying with cash, they avoid further opportunity costs, $\delta+\tau$, by holding the "worse" payment system. The surplus of sellers, $\psi_c(Q_c)$, on the other hand, decreases by accepting cash. As cash holders, they have "new" opportunity costs, $\delta+\tau$. Buyers as well as sellers know about that fact. As a consequence, sellers produce a smaller amount of goods, while buyers will accept the smaller amount of goods to get rid of cash. All in all, $\psi_c(Q_c)/\phi_c(Q_c)$ increases and the rhs of Figure 3.2 shifts to the left. In equilibrium, the traded amount of goods is smaller, $Q_c < Q$. If $Q < Q^*$ $(\rho > 1)$, a CBDC increases inefficiencies further. On opposite, if $Q > Q^*$ $(\rho < 1)$, a CBDC mitigates inefficiencies. In this case, the goods market in an SR is too competitive and the liquidity effect reduces the share of sellers. As a consequence, sellers produce less and the traded quantity converges to the optimum, $Q^* < Q_c < Q$. But there is a specific point where inefficiencies increase again. If the money supply increases further, the traded quantity, Q_c, decreases even more until $Q_c < Q^* < Q$. If the liquidity effect is too large, $Q^* - Q_c > Q - Q^*$ holds and a CBDC reinforces inefficiencies.

In a similar vein, if the amount of money, $1 - \mu_s$, increases, sellers are in a better position and produce a smaller amount of goods for one monetary unit. Thus, if money supply increases, prices also do. This implies that an increasing money supply increases (mitigates) inefficiencies in transactions with cash if $\rho > 1$ ($\rho < 1$).

Proposition 3.3 *If $\rho > 1$, a CBDC reinforces inefficiencies in transactions with cash in a DR by increasing money supply and opportunity costs for cash holders. If the goods market in an SR is too competitive, on the other hand, $\rho < 1$, a CBDC mitigates inefficiencies.*

3.4 Welfare Analysis

As proved above, a CBDC can reinforce as well as mitigate inefficiencies in transactions with cash in a DR. To answer how a CBDC affects welfare, I compare welfare between an SR and a DR. Again, welfare is given by average utility,

$$rW^{DR} = \mu_s[\mu_c\Delta(Q_c) + \mu_d\Delta(Q_d)] + \mu_c\gamma_c + \mu_d\gamma_d.$$

Welfare is given by the sum of the trade probabilities times the overall trade surplus plus the sum of the weighted monetary benefits. Welfare increases if $g(\lambda) \equiv rW^{DR} - rW^{SR} > 0$. Implementing $\mu_s = (1-\lambda)(1-\mu)$, $\mu_c = (1-\lambda)\mu$ and $\mu_d = \lambda$ in W^{DR} and rearranging yields

$$g(\lambda) = (1-\lambda)(1-\mu)[(1-\lambda)\mu\Delta(Q_c)+\lambda\Delta(Q_d)]+(1-\lambda)\mu\gamma_c+\lambda\gamma_d-\mu(1-\mu)\Delta(Q)-\mu\gamma_c.$$

I am not able to show that the sign of $g(\lambda)$ is the same for all parameter constellations. Therefore, I focus on the special case of a gradual introduction of digital money, only a small fraction of agents switches to a CBDC. I put it to the extreme and consider $g(\lambda)$ at $\lambda = 0$.

A first result does not surprise. For $\lambda = 0$, I get $g(0) = 0$. If there are no CBDC holders, the DR regime coincides with the SR regime. In a next step, I compute $g'(\lambda)$ and evaluate this function at $\lambda = 0$. If $g'(0) > 0$ [$g'(0) < 0$], the first unit of digital money improves (lowers) welfare. The derivation turns out to be

$$g'(0) = \overbrace{(1-\mu)[\gamma_d - \mu\Delta(Q_c)]}^{(1)} + \overbrace{\mu(\gamma_d - \gamma_c)}^{(2)} + \overbrace{(1-\mu)[\Delta(Q_d) - \mu\Delta(Q_c)]}^{(3)} \\ + \underbrace{\mu(1-\mu)\frac{\partial\Delta(Q_c)}{\partial\lambda}}_{(4)}. \tag{3.11}$$

I have to distinguish between four effects:

(1) Some sellers switch to CBDC holders and get the monetary benefit γ_d. A fraction μ of these sellers would have made an exchange of goods for cash, they face opportunity costs $\Delta(Q_c)$.

(2) Some cash holders switch from cash to CBDC, they get γ_d and lose γ_c.

(3) Some sellers now realize an exchange of goods for CBDC, they get $\Delta(Q_d)$. Again, a fraction μ of these sellers would have made an exchange of goods for cash with $\Delta(Q_c)$ as opportunity costs.

(4) Some sellers exchange for cash before and after the introduction of a CBDC. These sellers will be affected by the decline in Q_c, see above. If in the initial equilibrium the traded quantity Q_c is below (above) the optimal quantity Q_c^*, these sellers will lose, $\frac{\partial\Delta(Q_c)}{\partial\lambda} < 0$, (gain, $\frac{\partial\Delta(Q_c)}{\partial\lambda} > 0$).

Even for the special case of $\lambda = 0$, the net effect of (1) to (4) is difficult to sign. However, if there is a liquidity shortage in an SR, $\mu \to 0$, welfare clearly increases, $g'(0)|_{\mu=0} = \gamma_d + \Delta(Q_d) > 0$. The emission of a CBDC increases the money supply and thus the trade probability, and, in addition, allows for a monetary benefit γ_d. Let us exclude the scenarios of a liquidity shortage or an excess liquidity by assuming $\mu = 1/2$. In this case, the condition for a welfare improvement, $g'(0)|_{\mu=0.5} > 0$, simplifies to

$$\Delta(Q_d) - \Delta(Q_c) + \delta + \gamma_d + \frac{1}{2}\frac{\partial \Delta(Q_c)}{\partial \lambda} > 0.$$

Suppose that in the initial equilibrium $\rho_c(Q_c) > \rho_d(Q_d) > 1$ holds. In this case, inefficiencies in trades with digital money are smaller, $\Delta(Q_d) - \Delta(Q_c)$ is positive. Moreover, an increasing money supply increases inefficiencies in trades with cash further, $\frac{\partial \Delta(Q_c)}{\partial \lambda} < 0$ holds. As long as the the impact of λ on $\Delta(Q_c)$ is small, $\frac{\partial \Delta(Q_c)}{\partial \lambda} < 2[\Delta(Q_d) - \Delta(Q_c) + \delta + \gamma_d]$, there will be a welfare improvement. But without further restrictions on the utility and cost functions, a decline in welfare cannot be ruled out. But note that a sufficiently large value for γ_d ensures a positive welfare effect.

Proposition 3.4 *(i) If there is a liquidity shortage in the initial equilibrium, the gradual introduction of a CBDC is welfare improving. (ii) If there is neither a liquidity shortage nor an excess liquidity, $\mu = 1/2$, a sufficiently large value of the monetary benefit γ_d ensures a welfare gain.*

3.5 Conclusion

The task in Section 3 was to investigate how a CBDC affects inefficiencies and welfare, even if there is no liquidity shortage in a single currency regime. To do this, the search model of Trejos and Wright (1995) was extended by a CBDC. First of all, I show that a monetary benefit provides an easy solution for overcoming the discount problem. In this case, there is no trade off between optimizing the traded quantity and maximizing the number of trades. Even if the goods market is less competitive, the welfare-maximizing quantity of goods is traded since sellers get compensated by a subsidy. Moreover, the Trejos-Wright environment is well suited for investigating whether a CBDC reinforces inefficiencies in transactions with cash. Since a CBDC increases money supply and causes opportunity costs for cash holders, inefficiencies in transactions with cash increase. Nevertheless, the gradual introduction of a CBDC increases welfare, for most plausible parameter constellations the gains of the "new" CBDC holders exceed the loss of the remaining sellers and cash holders.

Next to the central bank as provider of cash and CBDC, there are further competitors on the market for payment systems, e.g, retail banks, providing deposits as digital payment system, or miners, providing cryptocurrencies. Since the search models of Kiyotaki and Wright (1993) and Trejos and Wright (1995) deal only with sellers and one or two types of money traders, the search model of Lagos and Wright (2005) is used in Section 4. With the Lagos-Wright framework numerous groups can be modeled simultaneously, questions about the currency competition can be answered. In addition, if all groups are part of one environment, a comprehensive welfare analysis can be done.

4 CBDC as Competitor for Bank Deposits and Cryptocurrencies[7]

4.1 Introduction

An increasing number of digital payment systems such as cryptocurrencies and mobile applications, e.g., the Swedish mobile app Swish, enhance competition on the market for payment systems. If these technological innovations better match customer preferences than traditional payment options such as cash and bank deposits, they will be more than a short-run phenomenon. Market shares of cash and bank deposits may erode even in the long run. In this section, I consider a world, where the central bank responds to this development by the introduction of an interest-bearing central bank digital currency (CBDC). The interest rate may serve as a new lower bound for any currency. Does, as a consequence, cash disappear? How do retail banks adjust the interest rate for deposits to curb a deposit outflow toward CBDC and cryptocurrencies, how is the interest rate for loans affected? On the other hand, is the central bank able to destroy the business model of cryptocurrency miners, since miners are forced to lower transaction fees, can the central bank enforce the bankruptcy of miners? This section sheds light on these questions.

From a customer point of view, the choice of a payment system is no either or decision. Typically, customers use two or more payment systems simultaneously. Decisive features of a payment system are the acceptance rate, storage costs, the real rate of return, the degree of anonymity, transaction fees, payment speed and security, see, e.g., Bagnall et al. (2016) or Mancini-Griffoli et al. (2019). The comparative disadvantage of cash is a negative real rate of return and high trans-

[7]This section is based on: Fuchs, Max (2022): CBDC as competitor for bank deposits and cryptocurrencies. MAGKS Discussion Paper No. 10/22.

action costs, at least for large-value and long-distance payments. The comparative disadvantage of deposits is the low degree of anonymity and the low speed of (cross-border) transfers, see Bindseil and Pantelopoulos (2022). Digital payment systems aim to eliminate these weaknesses. However, both central banks as provider of cash and retail banks as provider of deposits did not sit back and wait. Central banks are exploring the pros and cons of a CBDC, see Sveriges Riksbank (2021), retail banks are improving the payment structure which speed up and simplifies transactions, see Bech and Hancock (2020) or Blocher et al. (2017).

Analyzing digital payment systems has become a cottage industry in monetary economics, for cryptocurrencies see Böhme et al. (2015) or John et al. (2022), for a CBDC see Meaning et al. (2021), for corporate currencies see Hanl (2022) or Zetzsche et al. (2021), for retail and wholesale payment systems see Bindseil and Pantelopoulos (2022) or Petralia et al. (2019). The interaction between different payment systems, however, is less well explored. The exceptions are restricted mostly to the analysis of just two payment systems. Bindseil et al. (2021) focus on the interaction between an interest-bearing CBDC and bank deposits. To reduce the probability of a bank run, the central bank should set an upper bound for CBDC deposits. Chiu et al. (2021) show that the profit-maximizing response of a retail bank is an increase in the interest rate for bank deposits. Fernández-Villaverde and Sanches (2019) focus on the competition between privately-issued fiat monies. Schilling and Uhlig (2019a) model the competition between traditional cash and a privately-issued cryptocurrency.

But the investigation of repercussions and feedback effects requires an environment where a central bank (providing cash and CBDC), retail banks (providing deposits and loans) and miners (providing cryptocurrencies) act simultaneously. Since there is no work which includes all groups in one environment so far, I augment the search-theoretic model of Lagos and Wright (2005). In addition, with all groups, a comprehensive welfare analysis is possible. I show that the central bank is able to tilt the playing field until it wins by providing an interest-bearing CBDC. In this case, retail banks and miners are forced to match the CBDC rate to avoid runs. Retail banks will do this by a mixture of higher deposit rates and a decline in profits. Miners, however, go bankrupt since they are not able to offer such conditions. Alongside that, the crowding out of miners can be welfare-improving.

This section is organized into five parts. Section 4.2 reviews the literature, in particular, the literature dealing with digital payment systems as part of monetary search models. Section 4.3 describes the framework. Section 4.4 analyzes how a CBDC affects the business of retail banks and miners. A welfare analysis is done in Section 4.5. Section 4.6 concludes.

4.2 Literature Review

As Fernández-Villaverde (2018) already argued, to talk about money means to talk about trading frictions, the former exists because of the latter. Different from cash-in-advance or money-in-utility-function models, money has an explicit role in monetary search models: namely, to simplify trade by reducing trade frictions. Some of the monetary search models, in particular, models with digital money, are briefly mentioned in the following.

Lagos and Wright (2005, henceforth LW) set the stage for monetary search models and focus on the effects of discounting and bargaining power on consumption. Both issues cause inefficiencies so that the first best allocation is not reached. If tomorrow's consumption needs present-day money and money is discounted, private agents choose less money than necessary to buy the welfare-maximizing quantity of goods. Similarly, if the seller of a good has some power in the bargain over the price, the price exceeds costs of production. Thus, the first best allocation is reached only if there is no discounting and buyers have the complete bargaining power.

Chiu et al. (2021) use the LW framework to study the effects of an interest-bearing CBDC on retail banks. If retail banks have no market power, issuing an interest-bearing CBDC would crowd out retail banks. If retail banks have some market power, on the other hand, issuing an interest-bearing CBDC forces retail banks to increase their deposit rate to keep their customers. In this case, retail banks are able to finance the higher interest rate due to their profits. As a consequence, monopoly profits and inefficiencies decrease, consumption and welfare increase.

Fernández-Villaverde and Sanches (2019) implement cryptocurrencies in LW to investigate the competition between privately-issued cryptocurrencies. If the marginal costs of issuing new coins are zero, there will be no competitive equilibrium. Extending the model by government money ensures an equilibrium, but any equilibrium with private money is inefficient. In this way, the portfolio of private and government monies has a positive (negative) return if the overall money supply is shrinking (growing). As long as private agents value private cryptocurrencies, a government fails to implement the Friedman rule since private miners do not retract their previously issued coins. Instead, they will issue further coins so that the government is unable to reach its overall money supply target.

Davoodalhosseini (2021) also implements a CBDC in LW to distinguish between a cash-only, a CBDC-only and a cash-CBDC economy. The main advantage of cash is that it is anonymous. The main advantage of a CBDC is that it is interest-bearing.

As long as the (anonymous) costs for using a CBDC do not exceed a well-defined threshold, the CBDC-only scenario is welfare-maximizing. Yu (2022) studies the coexistence of cryptocurrencies and fiat money with the LW framework and shows that cryptocurrencies restrict the government's ability to overissue fiat money.

Almosova (2018a) uses LW to show that cryptocurrencies are able to set an upper limit for inflation if costs for emission of cryptocurrencies are manageable. Moreover, Almosova (2018b) uses LW to demonstrate that market tightness, defined as ratio between demanded transactions and miners, affects trade probability negatively. In this case, money demand is hump shaped: if the return for (providing) money is sufficiently high, there are enough miners and precautionary money demand decreases.

There is also some non-search-theoretic literature on a CBDC. Barrdear and Kumhof (2021) attest that a CBDC can raise GDP up to three percent due to a more efficient process of financial intermediation. Bordo (2021) points to some more benefits, firstly, the effective lower bound can be eliminated, secondly, price stability can be reached more easily, and thirdly, a CBDC can facilitate international transactions. The ECB (2020) emphasizes that a CBDC, which complements cash, could be necessary to secure demand for legal payment options. Nevertheless, there is no incentive to crowd out private solutions for efficient digital retail payments.

Alongside these, there is some literature on the impact on retail banks. Agur et al. (2022) distinguish between cash, deposits and a CBDC. If a CBDC is closely related to cash, there is the danger that cash will disappear. If a CBDC is similar to deposits, on the other hand, maturity transformation of commercial banks is at risk. Thus, there is a trade-off for the central bank: either cash or commercial banks are endangered. Andolfatto (2021) assesses the impact of an interest-bearing CBDC on a monopolistic retail bank sector and emphasizes that the introduction of a CBDC increases competition. Since the retail bank sector has to offer a higher deposit rate to keep its deposits, profits decrease.

Bindseil (2020) attests that the benefits of a CBDC include a more efficient retail payment system and a stronger monetary policy. Risks include, in particular, retail bank runs in crisis situations. In this case, a two-tier remuneration of a CBDC minimizes that risk. Chiu and Davoodalhosseini (2021) distinguish between two CBDC types: a cash-like type (non-interest-bearing) and a deposit-like type (interest-bearing). Depending on the type, the effects on welfare and bank intermediation differ. A cash-like CBDC is more able to promote consumption and thus welfare. Additionally, even in the absence of bank market power, a cash-like CBDC is able to increase bank intermediation by 5.8%. A deposit-like CBDC, on the other hand, promotes bank disintermediation by 2.6%.

Fernández-Villaverde et al. (2021) show with a Diamond and Dybvig (1983) model that the central bank can become the deposit monopolist by providing a CBDC. This endangers maturity transformation of retail banks. Kumhof and Noone (2018) attest that risks for retail bank runs are manageable, at least as long as some core principles for a CBDC are fulfilled. The core principles include an adjustable interest rate for a CBDC and a limited acquisition, i.e., a CBDC can only be acquired in exchange for government bonds at the central bank. Williamson (2021) studies the effects of a CBDC, which replaces cash, on financial stability and economic welfare. As long as transactions with a CBDC are more convenient, the probability of bank runs increases. Nevertheless, economic welfare can be higher since the gain from a CBDC exceeds the loss that occurs due to financial instability.

4.3 Framework

4.3.1 Environment

The framework is a combination of LW, Chiu et al. (2021) and Fernández-Villaverde and Sanches (2019). In contrast to the existing literature, the framework has two crucial innovations: firstly, there is a transaction fee which affects money demand, and secondly, the central bank, retail banks and miners are modeled simultaneously in one environment. If the transaction fee affects money demand, two payment systems do not necessarily need the same return to be used simultaneously in equilibrium. As a consequence, the choice of a payment system and thus the currency competition can be modeled in a better way. In addition, if all groups act in one environment, a more comprehensive welfare analysis is possible since gains and losses are possibly neutralized, e.g., the seigniorage of the central bank and the loss in purchasing power of private agents.

Each period is divided into two sub-periods, day and night. The discount factor between two periods is $\beta \in (0, 1)$. In the day, there is a decentralized bilateral matching market (DM) where only private agents act. At night, there is a centralized market (NM) where retail banks, entrepreneurs, miners and the central bank also act. On DM *special* goods are consumed where price and quantity are determined in a Nash bargaining process. Agents of type $j \in \{1, ..., J\}$ with $J \geq 3$, prefer special goods of type j but produce goods of type $j + 1$ (modulo J), see also Matsuyama et al. (1993). Thus, nobody consumes its own production and pure barter does not take place. This implies that money is necessary for trade on DM. In contrast, on NM a *general* good is consumed by everyone, the price of the general good is normalized to one. Before considering each group in detail, the overall environment is explained in brief.

The environment consists of private agents, retail banks, entrepreneurs, miners and the central bank, who are all connected through different transactions, see Figure 4.1.

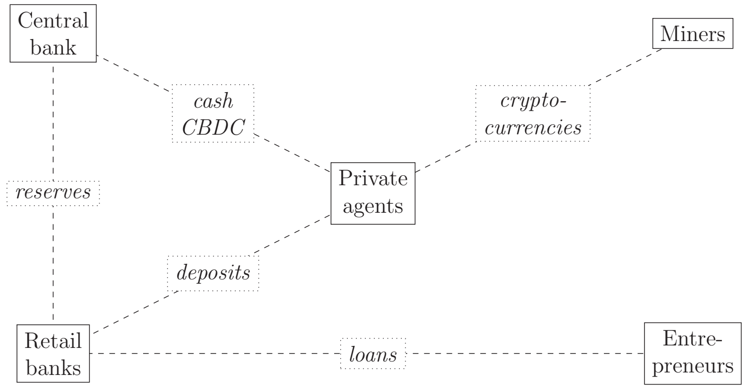

Figure 4.1: The Environment

Before entering DM, *private agents* hold an identical portfolio of cash ($i = 1$), CBDC ($i = 2$), deposits ($i = 3$) and cryptocurrencies ($i = 4$) to purchase special goods on DM. Note that only one payment system is used in a transaction, a mixture is impossible. Afterwards, the money that is left over from DM is used to finance the general good on NM and the portfolio for the next period on DM. Since all agents face the same maximization problem, the portfolio for the next period on DM is identical again. *Retail banks* use deposits from private agents for the loan business and the minimum reserve at the central bank. Only retail banks are able to offer loans to entrepreneurs since they are the only group that is able to reclaim the money. The profit from the loan business is used for the general good on NM. *Entrepreneurs* use loans from retail banks as investment capital. Different from the residual groups, entrepreneurs have the knowledge to multiply resources through investing. The gains are used to finance the general good on NM.

Miners provide cryptocurrencies as a payment system for private agents. Analogous to retail banks and entrepreneurs, the profit from mining is used to finance the general good on NM. Finally, the *central bank* offers cash and a CBDC as payment options for private agents and charges a minimum reserve from retail banks. The main goal of the central bank is to ensure the payment infrastructure. This requires a sufficient demand for legal payment options, i.e., cash and CBDC.

4.3.2 Private Agents

As in LW, there is a [0,1]-continuum of private agents acting either as a buyer or seller. The Bellman Equation for DM is

$$\begin{aligned} D(m) = \sum_{i=1}^{4} \underbrace{\alpha_i[N(m_i - p_i - \eta_i q_i, m_{-i}) + u(q_i)]}_{\text{purchase on DM with } i} \\ + \sum_{i=1}^{4} \underbrace{\alpha_i[N(m_i + p_i, m_{-i}) - c(q_i)]}_{\text{sale on DM for } i} + \underbrace{(1 - 2\sum_{i=1}^{4} \alpha_i) N(m)}_{\text{no trade on DM}}, \end{aligned} \tag{4.1}$$

where $m = (m_1, m_2, m_3, m_4)$ is the identical portfolio in the initial stage, with $m_i \equiv \phi_i m_i^n$ as real balances of payment system i, meaning the purchasing power per unit of i, ϕ_i, times the number of nominal units, m_i^n.

The first part of Equation (4.1) describes a purchase on DM, where $\alpha_i \equiv \tilde{\alpha}_i / J$ is the probability that payment system i is used by agents of type j. Here, $\tilde{\alpha}_i$ is the exogenous market share of payment system i with $\sum_{i=1}^{4} \tilde{\alpha}_i = 1$. Private agents take the market share as given, regardless of whether they use the payment system or not. Buyers have utility, $u(q_i) = \frac{q_i^{1-\sigma}}{1-\sigma}$, of consuming q_i with $\sigma \in (0, 1)$. They pay $p_i + \eta_i q_i$ and enter NM with $m = (m_i - p_i - \eta_i q_i, m_{-i})$, where $p_i \equiv \phi_i p_i^n$ is the real price, $\eta_i \equiv \phi_i \eta_i^n$ the real transaction fee and m_{-i} the real balances of the residual payment systems.

The transaction fee, $\eta_i \in [0, 1)$, can be a real transaction fee for the confirmation of a transaction as well as an anonymous cost if payment system i is not fully anonymous. It is assumed that the transaction costs increase in the transferred amount. Otherwise, the transaction fee does not affect money demand. This is an extension compared to the existing literature to model the costs and the degree of anonymity of a payment system. For instance, for small transactions with cash, there is no fee or anonymity cost. For transactions with a CBDC, deposits and cryptocurrencies, on the other hand, there can be a processing fee. Apart from cryptocurrencies, there are also anonymity costs if agents pay with a CBDC and deposits.

The second summand of Equation (4.1) covers a sale on DM. Sellers have costs, $c(q_i) = q_i$, for producing quantity q_i. They receive p_i and enter NM with $m = (m_i + p_i, m_{-i})$. Finally, the third part of Equation (4.1) describes the case where agents do not trade on DM so that they enter NM with the portfolio from the initial stage. Thus, for the NM the Bellman Equation is

$$N(m) = x_P + \beta D(m^+) \quad \text{with} \quad \underbrace{\sum_{i=1}^{4} m_i}_{\text{assets}} = \underbrace{x_P + \sum_{i=1}^{4} \psi_i m_i^+}_{\text{liabilities}}. \tag{4.2}$$

On NM agents consume and produce a general good, where x_P is the net consumption of private agents, the difference between one unit of the general good and one unit of work with a wage of one. Thus, on NM the utility and cost function are both linear with a slope of one. Without the quasi-linearity of, at least, one function, money demand for the next period is affected by the current money holdings and the cumulative distribution function (CDF) of money does not degenerate. Afterwards, they enter DM next period with m^+.[8] If $x_P = 0$, consumption equals wage and agents transfer their complete money $\sum_{i=1}^{4} m_i = \sum_{i=1}^{4} \psi_i m_i^+$ into the next period.

[8]All variables with a superscripted plus embody the *next* time period.

Here, $\psi_i \equiv \phi_i/\phi_i^+ \geq \beta$ is the price of payment system i, while $1/\psi_i$ is the return. Since ϕ_i is the purchasing power per unit, $1/\phi_i$ is equal to the price level. This implies that ψ_i is equal to the inflation rate. One can prove that $N(m_i, m_{-i})$ is linear in m_i by implementing the budget constraint in the Bellman Equation for NM. Using this and combining Equation (4.1) and (4.2) yields

$$D(m) = \sum_{i=1}^{4} \alpha_i \underbrace{[u(q_i) - p_i - \eta_i q_i]}_{\substack{\text{trade surplus}\\ \text{for a purchase}\\ \text{on DM}}} + \sum_{i=1}^{4} \alpha_i \underbrace{[p_i - c(q_i)]}_{\substack{\text{trade surplus}\\ \text{for a sale}\\ \text{on DM}}} + \sum_{i=1}^{4} \underbrace{(m_i - \psi_i m_i^+)}_{\substack{\text{net}\\ \text{consumption}\\ \text{on NM}}} + \beta D(m^+). \tag{4.3}$$

Agents' benefit consists of a surplus from a purchase on DM (utility of consumption minus price and transaction fee), a surplus from a sale on DM (price minus costs for production), net consumption on NM (current minus future money holdings) and the discounted utility from m^+. To determine q_i, the Nash bargaining product, defined by the product of a buyer's and seller's surplus of trading, is considered:

$$\max_{q_i, p_i} [u(q_i) - p_i - \eta_i q_i]^\theta [p_i - c(q_i)]^{1-\theta}.$$

To keep things tractable, it is assumed that buyers have the complete bargaining power, $\theta = 1$, and make a take-it-or-leave-it offer. This is analogous to Chiu et al. (2021) or Fernández-Villaverde and Sanches (2019) and excludes inefficiencies due to the bargaining power, inefficiencies arise only from discounting. As a consequence, buyers choose the quantity where sellers are indifferent between selling or not and offer a price which covers costs so that $p_i = c(q_i)$. In this way, they maximize their trade surplus, $\Delta_i(q_i) \equiv u(q_i) - p_i - \eta_i q_i > 0$. But the welfare-maximizing quantity, q_i^*, which is defined by $u'(q_i^*) = c'(q_i^*) + \eta_i$, is traded only if the buyers' money holdings are sufficiently large, $m_i \geq c(q_i^*)$. Otherwise, sellers are not willing to produce q_i^* because they are not fully compensated for their costs.

To obtain information about money holdings the FOC of Equation (4.3) regarding m_i^+ is considered. By doing so, $q_i^+ = p_i^+ = m_i^+$ holds. The first part, $q_i^+ = p_i^+$, holds due to the take-it-or-leave-it offer. Moreover, since money gets discounted, agents do not choose more money than necessary to pay the price, p_i^+, so that $p_i^+ = m_i^+$ holds. The inverse money demand function is

$$\Omega_i(m_i^+) = \underbrace{\alpha_i \left[\frac{1}{m_i^{+\sigma}} - (1+\eta_i) \right]}_{\substack{\text{equal to} \\ u'(q_i^+) - [c'(q_i^+) + \eta_i]}} \quad \Leftrightarrow \quad m_i^+ = \left[\frac{1}{\frac{\Omega_i}{\alpha_i} + (1+\eta_i)} \right]^{\frac{1}{\sigma}}, \tag{4.4}$$

where $\Omega_i(m_i^+) \equiv \frac{\psi_i(m_i^+)}{\beta} - 1 \geq 0$ are the storage costs of payment system i. Thus, $\Omega_i = 0$ implies $u'(q_i^+) = c'(q_i^+) + \eta_i$ so that the welfare-maximizing quantity, $q_i^{+*} = \sqrt[\sigma]{\frac{1}{1+\eta_i}}$, is traded. If $\Omega_i > 0$, on the other hand, there are storage costs for transferring money into the next period and agents do not demand the welfare-maximizing amount of money which is necessary to buy q_i^{+*}, the return, $1/\psi_i$, is too low. This also reveals two differences to the previous generation of Trejos and Wright (1995): firstly, the discount problem occurs on the buyer's side, and secondly, inefficiencies still evoke even if buyers have the complete bargaining power.

Money demand increases with the trade probability and decreases with the storage costs and transaction fee, see Figure 4.2. An increase in the trade probability causes a rotation to the right. An increase in the transaction fee causes a shift to the left. In both cases money demand is less elastic in the opportunity costs. Moreover, for a specific opportunity cost level, $\tilde{\Omega}_i$, money demand is the highest if the trade probability is high, $\alpha_i \to 1/3$, and the transaction fee low, $\eta_i = 0$, see the solid line. In addition, there are different parameter combinations where a specific amount of money, $\tilde{m}_i^+$, is demanded, e.g., if the storage costs and trade probability are high (low) and the transaction fee low (high), see the solid (dash-dotted dashed) line.

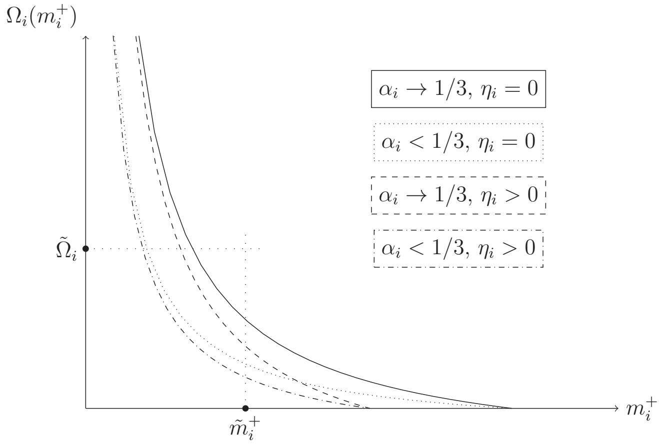

Figure 4.2: Money Demand

Interestingly, the trade probability (transaction fee) is decisive for money demand if the storage costs are high (low). Money demand functions with the same trade probability (transaction fee) converge if the storage costs increase (decrease). For instance, the solid and dashed line converge if the storage costs increase since both money demand functions have the same trade probability. If the storage costs are high, private agents are primarily concerned about the trade probability since a low trade probability implies a long holding duration and thus a high loss in purchasing power. If the storage costs are low, on the other hand, private agents face only a small loss in purchasing power so that they are primarily concerned about the transaction fee.

Proposition 4.1 *(i) The welfare-maximizing quantity, q_i^*, will be traded if and only if there are no storage costs, $\psi_i = \beta$. (ii) For $\psi_i > \beta$, agents' money demand is not sufficient to buy q_i^*, the traded quantity is lower, $q_i < q_i^*$.*

Equation (4.4) reveals that the CDF of money degenerates at the latest after the first period. Agents have different money holdings after DM, but they all face the same storage costs, trade probabilities and transaction fees. Due to the assumption of a linear cost function on NM, each agent chooses the same m^+. Each m_i^+ is unique since $\Omega_i'(m_i^+) < 0$ for all $m_i^+ > 0$.

Because of $u'(0) = \infty$, the expected value of money is always positive. This implies that at least one payment system will be used. Rearranging the expected utility and cost

$$\underbrace{\sum_{t=1}^{\infty} \overbrace{\left(\frac{1-\alpha_i}{1+\Omega_i}\right)^{t-1}}^{\substack{\text{prob. for no} \\ \text{trade until } t-1}} \overbrace{\left(\frac{\alpha_i}{1+\Omega_i}\right)}^{\substack{\text{prob. for a} \\ \text{trade in } t}} \overbrace{\left(\frac{m_i^{1-\sigma}}{1-\sigma} - \eta_i m_i\right)}^{\text{net utility}}}_{\text{discounted exp. net utility from consumption in period } t} > \underbrace{m_i}_{\substack{\text{exp. costs} \\ \text{(for sure)}}}$$

yields $\frac{m_i \Omega'(m_i)}{1-\sigma} < 0$, which, due to the assumption $\sigma \in (0,1)$, always holds. However, private agents use only the most economical payment systems. Using Equation (4.4), it can be shown that agents use only payment system i, $m_i > m_{-i} = 0$, if the overall cost difference

$$\Lambda_i^{-i} \equiv \left(\frac{\Omega_{-i}}{\alpha_{-i}} + \eta_{-i}\right) - \left(\frac{\Omega_i}{\alpha_i} + \eta_i\right)$$

between $-i$ and i is positive, $\Lambda_i^{-i} > 0$. Even if the residual payment systems $-i$ have a positive expected value, they are not used in equilibrium since the expected value is below the expected value of payment system i. If $\Lambda_i^{-i} = 0$, on the other hand, all payment systems are used simultaneously in equilibrium, $m_i = m_{-i} > 0$. Due to the transaction fee, i and $-i$ do not need the same storage costs to be used simultaneously. This is an extension compared to the previous literature, e.g., Chiu et al. (2021) or Fernández-Villaverde and Sanches (2019), where all payment systems must always have the same storage costs to be used simultaneously.

It is conceivable that payment system i has higher weighted storage costs but a lower transaction fee compared to the residual payment systems $-i$. For instance, cash and a CBDC are used simultaneously, $m_1 = m_2 > 0$, if and only if

$$\Lambda_2^1 = \underbrace{\left(\frac{\Omega_1}{\alpha_1} - \frac{\Omega_2}{\alpha_2}\right)}_{\substack{\text{weighted stor. cost} \\ \text{excess for cash}}} - \underbrace{(\eta_2 - \eta_1)}_{\substack{\text{anonymity cost} \\ \text{excess for a CBDC}}} = 0.$$

In this case, the weighted storage costs for cash are higher since cash is not interest-bearing. On the other hand, the transaction fee for a CBDC is higher since a CBDC is not fully anonymous.

Proposition 4.2 *Before entering DM, money holdings are homogeneous across all agents. (i) Because of $u'(0) = \infty$, agents choose at least one payment system in equilibrium. (ii) If $\Lambda_i^{-i} > 0$, agents use only payment system i. If $\Lambda_i^{-i} = 0$, on the other hand, agents use all four payment systems simultaneously.*

4.3.3 Retail Banks

Next to private agents, there is a finite number of retail banks using their loan business to ensure net consumption, x_B, on NM, see also Chiu et al. (2021). Since only retail banks are able to reclaim loan payments, they are the only group that is able to offer loans to entrepreneurs. Retail banks use deposits, $\psi_3 m_3$, from private agents for two assets: a share of $\chi \in (0, 1)$ must be used as a reserve which is deposited at the central bank. The interest rate for reserves is equal to the interest rate of cash, $1/\psi_1$. Thus, one can also argue that retail banks have to hold a cash reserve. The residual share, $1 - \chi$, is used for loans given to entrepreneurs where the loan rate is $\rho > 1$. Hence, the weighted return is $\Gamma \equiv \chi/\psi_1 + (1 - \chi)\rho > 0$.

Revenues from deposits, $\Gamma\psi_3 m_3$, and transaction fees, $\alpha_3\eta_3 m_3$, are used to finance net consumption, x_B, on NM and, at the end of the period, pay back deposits, m_3. Since net consumption is equal to the profit, retail banks maximize their profit subject to their budget constraint

$$\underbrace{\Gamma\psi_3 m_3 + \alpha_3\eta_3 m_3}_{\text{assets}} = \underbrace{x_B + m_3}_{\text{liabilities}}. \tag{4.5}$$

As in Chiu et al. (2021) and due to empirical evidence of Dreschler et al. (2017), it is assumed that the deposit market is non-competitive while the loan market is competitive. Thus, retail banks maximize their profit by choosing their deposits, m_3. With respect to Equation (4.4), the FOC is

$$\psi_3(m_3) + m_3\psi_3'(m_3) = \frac{1-\alpha_3\eta_3}{\Gamma}. \tag{4.6}$$

Since $\psi_3(m_3) + m_3\psi_3'(m_3)$ is decreasing in m_3, retail banks increase their deposits if the trade probability, transaction fee or weighted return increases.

Next to the deposit channel there is also the loan channel. Loan supply, ℓ_s, depends on the interest rate for reserves and loans. Three cases are possible:

- If $\rho < 1/\psi_1$, loan supply is zero. In this case, there is no incentive for retail banks to invest in loans since the interest rate for reserves is higher.
- If $\rho = 1/\psi_1$, loan supply is between zero and $(1-\chi)\psi_3 m_3$. Since the interest rate for reserves and loans are equal, retail banks are indifferent about investing in reserves or loans. As long as $\ell_s < (1-\chi)\psi_3 m_3$, retail banks also hold an excess reserve.
- If $\rho > 1/\psi_1$, loan supply is $(1-\chi)\psi_3 m_3$. The loan rate exceeds the interest rate for reserves and retail banks invest all their remaining deposits in loans.

Loans do not necessarily increase in ρ. If ρ increases, deposits, m_3, do too, see Equation (4.6). But ψ_3 is decreasing in m_3, see Equation (4.4). Thus, it is not guaranteed that $m_3\psi_3(m_3)$ is increasing in m_3. Observing Equation (4.4), it is straightforward to show that the condition

$$\eta_3 < \frac{1-\alpha_3}{\alpha_3} + \frac{1-\sigma}{m_3^\sigma} \tag{4.7}$$

secures that $m_3\psi_3(m_3)$ is increasing in m_3 so that ℓ_s is increasing in ρ. Here, $\eta_3 \in [0, 1)$ is sufficient for the validity of Condition (4.7) since $\frac{1-\alpha_3}{\alpha_3} \geq 2$ and $\frac{1-\sigma}{m_3^\sigma} > 0$. Condition (4.7) extends Chiu et al. (2021) who focus on the case without a transaction fee, $\eta_3 = 0$. If the trade probability and transaction fee are too large, money demand is less elastic in the opportunity costs, meaning ψ_3 decreases significantly for a small increase in m_3. In this area $m_3\psi_3(m_3)$ is decreasing in m_3 and loan supply is decreasing in the loan rate.

Proposition 4.3 *(i) If the interest rate for loans exceeds the interest rate for reserves, retail banks offer loans to entrepreneurs. (ii) The loan supply increases in the loan rate if the transaction fee for deposits, η_3, does not exceed the threshold given by Condition* (4.7).

4.3.4 Entrepreneurs

The customers for the loans are a continuum of entrepreneurs. I follow Chiu et al. (2021) and assume that entrepreneurs have an investment opportunity to transform a unit of ℓ into $f(\ell) = \frac{\ell^{1-\varepsilon}}{1-\varepsilon}$ units of ℓ, with $\varepsilon \in (0, 1)$ as investment efficiency factor. In order to finance the investments, entrepreneurs demand loans. Entrepreneurs maximize their net consumption, x_E, subject to their budget constraint

$$x_E = f(\ell) - \rho\ell. \tag{4.8}$$

The solution to maximize profit, delivers the loan demand curve

$$\ell_d = (1/\rho)^{1/\varepsilon}.$$

Demand is decreasing in the loan rate, ρ, and increasing in the investment efficiency factor, ε. If the investment efficiency factor increases, demand shifts to the right.

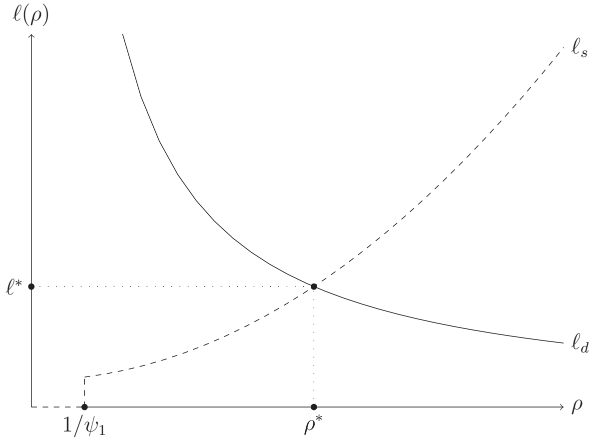

Figure 4.3: The Loan Market

The dashed curve captures loan supply, ℓ_s. As mentioned above, loan supply is zero as long as the interest rate for reserves exceeds the loan rate. If the rates are equal, retail banks are indifferent and loan supply is between zero and $(1-\chi)\psi_3 m_3$. Finally, if the loan rate is higher, retail banks invest all their remaining deposits in loans so that $\ell_s = (1-\chi)\psi_3 m_3$. In Figure 4.3, it is assumed that Condition (4.7) holds. Hence, loan supply is increasing in ρ.

4.3.5 Miners

Similar to retail banks, there is a limited number of miners providing the fourth payment method, cryptocurrencies, see also Fernández-Villaverde and Sanches (2019). It is assumed that either all miners provide the same cryptocurrency or each miner provides another one. If every miner provides another type, all agents hold a portfolio in which every cryptocurrency has the same weight. Miners act only on NM where they maximize their net consumption, x_M. Their earnings are given by revenues from mining and transaction fees. If real money balances are constant, the inflation rate, ψ_i, is equal to the money growth rate, see also LW. Thus, $\delta \equiv (\psi_4 - 1)m_4$ is equal to the emission of new coins within a period. Since the value of money is one within a period, δ is also equal to the revenues from mining.

Moreover, miners have costs, $k(\delta)$, for issuing coins. The cost function, $k(\delta)$, satisfies $k(0) = 0$, $k'(0) = \kappa_t > 0$, $k'(\delta) > 0$ and $k''(\delta) \geq 0$. Since mining gets more difficult by the period, the emission of a new coin gets more costly by the period.[9] The linear cost parameter, $\kappa_t \equiv t\bar{\kappa}$, captures this feature, κ_t is assumed to increase over time. By contrast, if $k'(0) = 0$, money supply would be infinite, see also Fernández-Villaverde and Sanches (2019). Thus, miners maximize their profit subject to their budget constraint

$$x_M = \delta + \alpha_4 \eta_4 m_4 - k(\delta). \tag{4.9}$$

To illustrate, let us assume $k(\delta) = \frac{\delta^3}{3} + \kappa_t \delta$ so that $\delta^*(t) = \sqrt{1 - t\bar{\kappa}}$. Thus, miners do not issue further coins from $t \geq 1/\bar{\kappa}$ since mining is no longer profitable. From this point, they receive only the transaction fees. If $\bar{\kappa}$ is small (large), miners issue coins for numerous (a few) periods, see the dotted (solid) line in Figure 4.4.

[9] Usually, mining costs depend on the amount of coins that are already in circulation, see, e.g., Bitcoin. But this approach requires to model the overall amount of coins. Based on some own calculations, I conclude that the additional insights do not warrant the additional algebra, I give precedence to simplicity.

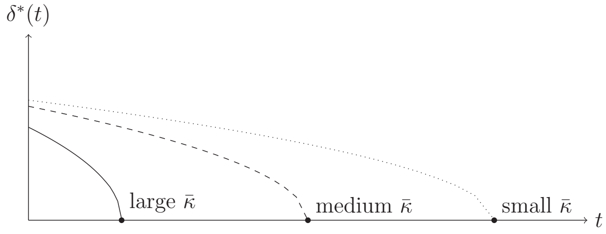

Figure 4.4: Issue of New Coins

4.3.6 Central Bank

In contrast to the previous literature, e.g., Chiu et al. (2021) or Fernández-Villaverde and Sanches (2019), I explicitly model the central bank here. Only in this case, interest payments are neutral with respect to welfare since the seigniorage of the central bank is equal to the loss in purchasing power of private agents. Nevertheless, the interest rate still affects consumption on DM and NM, see also Section 4.5. The central bank is not interested in maximizing its own net consumption, x_C. Instead, the main goal is to ensure the payment infrastructure, see, e.g., Article 3 of the ECB statute: "...promote the smooth operation of payment systems." (ECB, 2004). Fulfilling this task requires that the central bank is involved in a large fraction of payments, the demand for cash and CBDC has to be ensured. As long as cash is anonymous and has no fee, $\eta_1 = 0$, the budget constraint of the central bank reads

$$\underbrace{(\psi_1 - 1)\left[m_1 + \frac{\chi\psi_3 m_3}{\psi_1}\right] + \alpha_2\eta_2 m_2}_{\text{assets}} = \underbrace{x_C + (1-\psi_2)m_2}_{\text{liabilities}}. \tag{4.10}$$

If $\psi_1 > 1$, assets are given by the emission of cash, the reserve requirement and the transaction fee from a CBDC. Liabilities, on the other hand, are given by net consumption and the interest rate payment for a CBDC, at least if $\psi_2 < 1$.

4.4 Environment with(out) a CBDC

After considering the environment, I distinguish between two cases now: in case (A) the central bank has only cash available (Section 4.4.1). In case (B), on the other hand, the central bank has also a CBDC available (Section 4.4.2).

4.4.1 Environment without a CBDC

In case (A) the central bank has only cash available. To segregate cash from a CBDC, I assume that cash has no positive return, $\psi_1 \geq 1$. Otherwise, if $\psi_1 < 1$, cash is interest-bearing and the only additional monetary policy measure from a CBDC is given by adjusting the transaction fee, η_2. To secure demand for cash, the only option for the central bank is to stop the emission of further cash. In this case, the return for cash is at least zero. Nevertheless, there are storage costs since cash gets discounted by β. Thus, $\Omega_1 = \frac{1-\beta}{\beta} > 0$ holds and q_1^* is not traded. Since the Friedman rule cannot be implemented, the first best allocation is missed.

If the cost differences, Λ_3^1 and Λ_4^1, are positive for $\psi_1 > 0$ and negative for $\psi_1 = 0$, retail banks and miners have to adjust the storage cost and/or transaction fee of their payment systems as soon as the central bank stops the emission of further cash. Thus, they cannot maximize their profits by choosing m_3 and δ furthermore. Since trade probabilities are exogenous, there is no incentive to underbid the conditions for cash to possibly increase the market share. Both groups try to match the conditions for cash so that the overall cost difference is zero again, $\Lambda_3^1 = \Lambda_4^1 = 0$. In this case, cash, deposits and cryptocurrencies would be used simultaneously in equilibrium. To achieve $\Lambda_3^1 = \Lambda_4^1 = 0$, retail banks and miners have two options: either they charge no fee so that they are able to offer a lower interest rate or they offer a high interest rate so that they are able to charge a fee.

Retail banks are able to offer $1/\psi_3 = \frac{\Gamma}{1-\alpha_3\eta_3}$. In this case, they make zero profits and their cost term, $\frac{\Omega_3}{\alpha_3} + \eta_3$, is in its minimum. Implementing the maximal interest rate, $1/\psi_3 = \frac{\Gamma}{1-\alpha_3\eta_3}$, in the cost term yields $\left(\frac{1}{\alpha_3} - \eta_3\right)\left(\frac{1}{\beta\Gamma} - 1\right)$. Thus, for $\Gamma > 1/\beta$, retail banks charge no fee, $\eta_3 = 0$, since their cost term is smaller compared to the case where they charge a fee. Since the loan business is profitable, it makes sense to focus on it. For $\Gamma = 1/\beta$, retail banks are indifferent. Finally, for $\Gamma < 1/\beta$, retail banks charge a fee. Since the loan business is less profitable, it makes sense to focus on a fee for deposits.

Assuming $\Gamma = 1/\beta$ and $\eta_3 = 0$, retail banks have to offer an interest rate for deposits of

$$\Lambda_3^1 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_3} = \frac{1}{\beta + \alpha_3\left(\frac{1-\beta}{\alpha_1}\right)}.$$

Here, $\alpha_3 = \alpha_1$ implies $1/\psi_3 = 1$. If trade probabilities are equal, retail banks have to match the return for cash. If $\alpha_3 < \alpha_1$, retail banks have to offer a positive return to compensate private agents for the lower trade probability. Even if $\alpha_3 \to 0$, retail banks do not go bankrupt. In this case, they have to offer $1/\psi_3 = 1/\beta$, which is, due to the assumption of $\Gamma = 1/\beta$ and $\eta_3 = 0$, possible. Since $1/\psi_3 = 1/\beta$, there are no storage costs so that α_3 does not affect the cost term anymore. If $\alpha_3 > \alpha_1$, on the other hand, retail banks are able to offer a negative return.

The situation is similar for miners. Since the value for new issued coins is one within a period, miners get no interest payments on new issued coins. This is equal to $\Gamma = 1$, see Equation (4.5) and (4.9). Thus, miners always charge a fee and have to offer

$$\Lambda_4^1 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_4} = \frac{1}{\beta + \alpha_4\left(\frac{1-\beta}{\alpha_1} - \beta\eta_4\right)}.$$

As long as $\alpha_4 < \left[\frac{1}{\alpha_1} - \left(\frac{\beta}{1-\beta}\right)\eta_4\right]^{-1}$, miners have to offer a positive return on cryptocurrencies to compensate private agents for charging a fee by retracting their coins, $\delta < 0$. Miners make no losses as long as $x_M \geq 0$, which implies $\psi_4 \geq 1 - \alpha_4\eta_4$. Thus, miners are able to match the conditions for cash as long as

$$\frac{1}{\beta + \alpha_4\left(\frac{1-\beta}{\alpha_1} - \beta\eta_4\right)} \leq \frac{1}{1 - \alpha_4\eta_4} \quad \Leftrightarrow \quad \eta_4 \geq \frac{\alpha_1 - \alpha_4}{\alpha_1\alpha_4}.$$

Since $\psi_4 = 1 - \alpha_4\eta_4 \geq \beta$, there is no space for η_4 if $\alpha_4 < \beta\alpha_1$. In this case, miners go bankrupt since they are not able to compensate private agents for the low trading probability and the transaction fee for cryptocurrencies. Thus, $\alpha_4 \geq \beta\alpha_1$ turns out to be a necessary condition for an equilibrium with cryptocurrencies. The lower bound, $\beta\alpha_1$, increases with the discount rate: if agents value the future more, the interest rate is even more important.

4.4.2 Environment with a CBDC

In case (B) the central bank has a CBDC available. Since a CBDC is interest-bearing, there are more monetary policy measures for the central bank to ensure demand for legal payment options. In general, a CBDC is superior to cash if

$$\Lambda_2^1 > 0 \quad \Leftrightarrow \quad \frac{1}{\psi_2} > \frac{1}{\hat{\psi}_1} \equiv \frac{1}{\alpha_2\left[\beta\left(\frac{1}{\alpha_2} - \eta_2\right) + (1-\beta)\left(\frac{1}{\alpha_1}\right)\right]}.$$

Even if the central bank offers a fully anonymous CBDC without a transaction fee by choosing $\eta_2 = 0$, the central bank has to ensure that the return for a CBDC exceeds a lower bound, $1/\hat{\psi}_1$. Otherwise, the central bank has no higher power compared to the case without a CBDC. Note that private agents choose only CBDC instead of cash if the CBDC rate exceeds the lower bound, $1/\hat{\psi}_1$. In this case, a CBDC displaces cash.

As mentioned above, as long as $\Gamma = 1/\beta$, retail banks charge no fee and are able to offer $1/\psi_3 = 1/\beta$. Thus, retail banks are always able to match the conditions for a CBDC, even if $\eta_2 = 0$. Due to $\Lambda_3^2 = 0$, a CBDC and deposits are used simultaneously in equilibrium. Now, miners have to offer

$$\Lambda_4^2 = 0 \quad \Leftrightarrow \quad \frac{1}{\psi_4} = \frac{1}{\beta + \alpha_4 \left[\frac{\psi_2 - \beta}{\alpha_2} - \beta(\eta_4 - \eta_2)\right]},$$

while they are able to offer $1/\psi_4 = 1/(1 - \alpha_4\eta_4)$. Thus, miners are able to match the conditions for a CBDC as long as

$$\begin{aligned} & \frac{1}{\beta + \alpha_4 \left[\frac{\psi_2 - \beta}{\alpha_2} - \beta(\eta_4 - \eta_2)\right]} \leq \frac{1}{1 - \alpha_4\eta_4} \\ \Leftrightarrow \quad & \frac{1}{\psi_2} \leq \frac{1}{\hat{\psi}_4} \equiv \frac{1}{\alpha_2 \left[\beta \left(\frac{1}{\alpha_2} - \eta_2\right) + (1-\beta)\left(\frac{1}{\alpha_4} - \eta_4\right)\right]}. \end{aligned}$$

In general, this is possible since $1/\hat{\psi}_4 > 1/\hat{\psi}_1$ holds, at least for $\alpha_4 > \beta\alpha_1$, see also Section 4.4.1. If the central bank chooses $\eta_2 = 0$, the upper bound decreases to

$$\frac{1}{\hat{\psi}_4} = \frac{1}{\beta + (1-\beta)\left(\frac{\alpha_2}{\alpha_4} - \alpha_2\eta_4\right)}.$$

Since $(1-\beta)\left(\frac{\alpha_2}{\alpha_4} - \alpha_2\eta_4\right) > 0$, there is always a space for the central bank to drive miners out of the market. Thus, if the CBDC rate is inside

$$\frac{1}{\hat{\psi}_4} < \frac{1}{\psi_2} \leq \frac{1}{\beta},$$

miners go bankrupt since they are not able to compensate private agents for the transaction fee. All in all, the central bank has a higher power with a CBDC to displace private payment systems since a CBDC can be interest-bearing. Table 4.1 summarizes the results for case (A) and (B).

case	(A) only cash	(B) also CBDC
instrument for the central bank	money growth rate of zero ($\Omega_1 > 0$ still holds)	raising the CBDC rate ($\Omega_2 = 0$ is possible)
reference for retail banks and miners	conditions for cash ($\Lambda_3^1 = \Lambda_4^1 = 0$)	conditions for CBDC ($\Lambda_3^2 = \Lambda_4^2 = 0$)
payment systems used in equilibrium	if $\alpha_4 \geq \beta\alpha_1$, cash, deposits, cryptocurrencies	if $1/\psi_2 \in (1/\hat{\psi}_1, 1/\hat{\psi}_4]$, CBDC, deposits, cryptocurrencies
	if $\alpha_4 < \beta\alpha_1$, cash, deposits	if $1/\psi_2 \in (1/\hat{\psi}_4, 1/\beta]$, CBDC, deposits

Table 4.1: Case (A) and (B)

Proposition 4.4 *Assume that the loan business of retail banks is profitable, $\Gamma \geq 1/\beta$. (i) In case (A), cash, deposits and cryptocurrencies are used. For $\alpha_4 < \beta\alpha_1$, only cash and deposits are used. (ii) In case (B), cash is replaced by a CBDC. For $1/\psi_2 > 1/\hat{\psi}_4$, only a CBDC and deposits are used.*

4.5 Welfare Analysis

In the last step, it should be answered whether providing a CBDC improves welfare. Welfare is defined as the sum of consumption on DM and NM. Since the growth rates for cash and cryptocurrencies are zero in steady state, implementing the budget constraints (4.2), (4.5), (4.8), (4.9) and (4.10) in the welfare function yields

$$W = \sum_{t=0}^{\infty} \beta^t \left(\sum_{i=1}^{4} \alpha_i \Delta_i + x_P + x_B + x_E + x_M + x_C \right)$$

$$\Rightarrow \quad W = \frac{\sum_{i=1}^{4} \alpha_i \bar{\Delta}_i + f(\ell) - \ell}{1-\beta}.$$

Here, $\bar{\Delta}_i \equiv u(q_i) - c(q_i)$ is the net trade surplus without a transaction fee. Since transaction fees are a revenue as well as a cost for a specific group they do not affect welfare directly. Nevertheless, transaction fees affect consumption on DM and thus welfare in an indirect way. The argumentation is similar for the loan rate. If $\psi_i = \beta$ and $\eta_i = 0$, the net trade surplus is at its maximum, $\bar{\Delta}_i = \frac{\sigma}{1-\sigma}$. If the central bank, retail banks and miners were all able to offer such conditions, money holdings would be equal across all payment systems and welfare would be at its overall maximum,

$$W^* = \frac{\left(\frac{1}{J}\right)\left(\frac{\sigma}{1-\sigma}\right) + \frac{[(1-\chi)\beta]^{1-\varepsilon}}{1-\varepsilon} - (1-\chi)\beta}{1-\beta}.$$

But W^* cannot be reached since the central bank cannot implement $\psi_1 = \beta$ for cash, while miners are not able to offer $\psi_4 = \beta$ and $\eta_4 = 0$ simultaneously for cryptocurrencies. Thus, as long as $\psi_i = \beta$ and $\eta_i = 0$, the net trade surplus and the amount of loans are at their maximum but cash and cryptocurrencies are not used in equilibrium so that the trade probability is below $1/J$. If $\psi_i > \beta$ and/or $\eta_i > 0$, on the other hand, the trade probability, given by $1/J$, is maximal, but the net trade surplus and the amount of loans are less, $\bar{\Delta}_i < \frac{\sigma}{1-\sigma}$ and $\ell < (1-\chi)\beta$.

In the next step, welfare between case (A) and (B) should be compared. In both cases welfare is below W^*. In case (A) only cash is available. For $\alpha_4 \geq \beta\alpha_1$, retail banks and miners are able to match the conditions for cash so that $\Lambda_3^1 = \Lambda_4^1 = 0$. Thus, welfare is

$$W_A = \frac{(\alpha_1 + \alpha_3 + \alpha_4)\bar{\Delta}_A + f(\ell_A) - \ell_A}{1-\beta},$$

where $\bar{\Delta}_A$ and ℓ_A are the net trade surplus and the amount of loans in case (A). Since $\Lambda_3^1 = \Lambda_4^1 = 0$, money holdings and thus the net trade surplus are equal across all three payment systems.

In case (B) a CBDC is available. For $1/\psi_2 > 1/\hat{\psi}_4$, only retail banks are able to match the conditions for a CBDC, while miners go bankrupt, $\Lambda_3^2 = 0 > \Lambda_4^2$. As long as a CBDC and deposits are used simultaneously in equilibrium, welfare in case (B) is

$$W_B = \frac{(\alpha_2 + \alpha_3)\bar{\Delta}_B + f(\ell_B) - \ell_B}{1 - \beta}.$$

Since the central bank forces retail banks to increase the interest rate for deposits compared to case (A), demand for deposits and hence the net trade surplus in a trade with deposits increases, $\bar{\Delta}_B > \bar{\Delta}_A$. Moreover, since the amount of deposits increases, loans also do from ℓ_A to ℓ_B, see Figure 4.5.

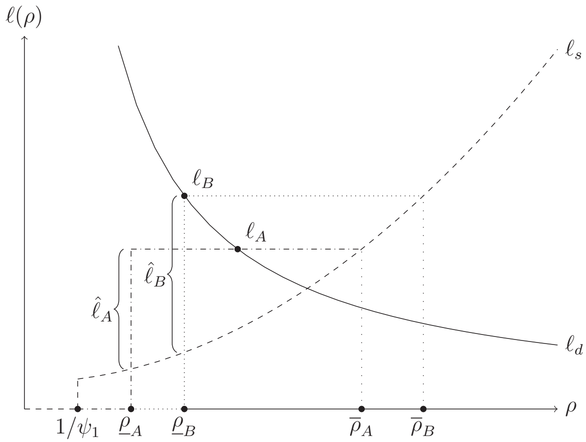

Figure 4.5: The Loan Market (with a CBDC)

If retail banks face further payment systems, they have to offer a certain deposit rate to ensure $\Lambda_3^{-i} \geq 0$ and need a minimum loan rate, $\underline{\rho}$, to offer the required deposit rate. Due to the higher deposit rate, deposits and loans increase for $\rho = \underline{\rho}$, see $\hat{\ell}$. For $\rho > \underline{\rho}$, retail banks make profits again. Since the amount of loans is still above the optimal amount of loans without further payment systems (dashed line), retail banks do not increase loan supply. Finally, for $\rho > \overline{\rho}$, loan supply increases since it is profit maximizing again (dash-dotted and dotted line merge with the dashed line). Note that the amount of loans is maximal for $\underline{\rho} = \underline{\rho}_B$. For $\underline{\rho} > \underline{\rho}_B$, the amount of loans decreases again.

On the one hand, consumption in a trade with deposits and gains from investment capital increase. On the other hand, as long as $\alpha_2 < \alpha_1 + \alpha_4$, the number of trades on DM decreases since only a CBDC and deposits are used in equilibrium. The gain exceeds the loss if $W_B > W_A$ which requires

$$\underbrace{\alpha_2 \bar{\Delta}_B}_{\substack{\text{trades with} \\ \text{a CBDC}}} + \underbrace{\alpha_3(\bar{\Delta}_B - \bar{\Delta}_A)}_{\substack{\text{add. trade surplus} \\ \text{for deposits}}} + \underbrace{[f(\ell_B) - \ell_B] - [f(\ell_A) - \ell_A]}_{\substack{\text{surplus from} \\ \text{investment capital}}} > \underbrace{(\alpha_1 + \alpha_4)\bar{\Delta}_A}_{\substack{\text{missed trades} \\ \text{with cash and} \\ \text{cryptocurrencies}}}. \tag{4.11}$$

As long as the trade probabilities for cash and cryptocurrencies are limited, Condition (4.11) is fulfilled for sure. In this case, the loss due to the missed trades with cash and cryptocurrencies is manageable since only a few trades disappear. If the trade probabilities are large, on the other hand, there is the risk that the loss exceeds the gain from using a CBDC. Thus, the sign of the welfare effect mainly depends on the circulation of the different payment systems. This result confirms the findings from Fuchs and Michaelis (2023) who emphasize that the welfare effect mainly depends on the fraction of agents using digital money.

Proposition 4.5 *For $1/\psi_2 > 1/\hat{\psi}_4$, consumption in a trade with a CBDC and deposits as well as gains from investment capital increases. But, as long as $\alpha_2 < \alpha_1 + \alpha_4$, the number of trades on DM decreases since trades with cash and cryptocurrencies disappear. Thus, welfare increases only if the gain exceeds the loss, Condition* (4.11) *has to be fulfilled.*

4.6 Conclusion

This section provides some useful insights about the competition between legal and private payment options. Firstly, the trade probability (transaction fee) is of importance for money demand if the storage costs are high (low). Secondly, even if every payment system has a positive expected net value, private agents use only the payment systems with the highest expected net value. Thirdly, loan supply only increases in the loan rate if money demand is elastic in the opportunity costs for holding money. Fourth, since mining gets more difficult over time, miners only issue coins up to a certain point in time. And fifth, and most important, the central bank is able to secure demand for legal payment options, and thus the payment infrastructure, by providing an interest-bearing CBDC. In this case, retail banks and miners are forced to match the conditions for a CBDC to avoid runs. Retail banks will do this. By contrast, miners go bankrupt. Alongside that, welfare increases if the gain due to the higher consumption and the higher amount of loans exceeds the loss due to fewer trades. Thus, the sign of the welfare effect mainly depends on the circulation of the different payment systems.

5 Conclusion

There is one thing for sure: digital payment systems are on the rise. They already have an economic impact, and the impact will be even stronger in the future. This development causes numerous questions: are digital payment systems able to improve welfare? Do they affect the value of cash? On the other hand, is a central bank able to ensure the payment infrastructure by issuing its own digital currency? How do retail banks and miners react? This work sheds light on these questions.

First of all, Section 2 shows that a partially accepted secondary (digital) currency, which circulates next to fully accepted cash, increases welfare only if the share of digital money traders replacing sellers is large. Thus, digital money and cash have to be complements. If the secondary currency is also fully accepted, on the other hand, welfare also increases if the share of digital money traders replacing sellers is small, both currencies can be substitutes.

Since a secondary currency does not only affect welfare, Section 3 investigates whether a secondary (digital) currency, i.e., a CBDC, affects the value of cash. As long as the share of CBDC holders and the paid interest on the CBDC are sufficiently large, the value of cash decreases. On the one hand, money supply and thus prices increase, on the other hand, opportunity costs for holding cash increase since cash is not interest-bearing.

Finally, Section 4 shows that a central bank is able to ensure the payment infrastructure by issuing an interest-bearing CBDC. In this case, retail banks and miners have to match the CBDC rate to avoid runs. Retail banks face lower profits but survive. Miners, on the other hand, go bankrupt.

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