Received: 7 July 2023 | Revised: 5 September 2023 | Accepted: 9 October 2023

DOI: 10.1002/pamm.202300258

PAMM Proceedings in Applied Mathematics and Mechanics

RESEARCH ARTICLE

Topology optimization considering self-weight

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Abstract
In civil engineering self-weight loading usually represents the preponderant load case and shall therefore be considered in structural optimization. The present study investigates numerical topology optimization of structural stiffness with a bound on disposable amount of material. A customized topology optimization routine based on Sequential Quadratic Programming was built and is examined by means of known benchmark problems complemented by consideration of self-weight. This study aims to highlight the relevance of consideration of self-weight in resource efficient structural design by demonstrating its impact on the optimal structural layout.

1 | INTRODUCTION

Topology optimization represents a valuable design tool for resource efficient structures that are characterized by a high aesthetic potential. Desired structural properties are achieved or improved by geometrical adaption of structural elements to the load. Topology optimization is usually carried out in terms of a material distribution problem in a predefined design domain with respect to a distinctive load profile. The most popular approaches to solve this problem were introduced in [1, 2]. The majority of publications in the field treats topology optimization with respect to constant external loading, for example [2, 3]. For applications in civil engineering, self-weight represents the preponderant load case which motivates to focus this study on topology optimization with respect to this particular design-dependent load. Master builders like Gaudí and Isler used hanging shapes to design their unique edifices [4, 5]. Hanging shapes are characterized by load transfer purely by normal forces and can therefore be considered optimal. Based on this experimental approach to form finding, the present study aims to demonstrate the necessity for consideration of self-weight in applications of topology optimization in civil engineering by investigation of the impact of self-weight on the layout of the optimized structure. Furthermore, it is exemplarily investigated whether the resulting structures can be considered as efficient from an engineering perspective. Homogeneous structural load as well as load transfer by normal forces are significant for evaluation. The popular benchmark problem optimization of structural stiffness with a bound on disposable material is considered. Widespread strategies for its solution are the Method of Moving Asymptotes (MMA), see [6], and Optimality Criteria Methods (OCM) [7]. Consideration of design dependent load impacts the properties of the investigated structural optimization problem and hampers treatment with the classic MMA [8] while the heuristic update scheme for the design variables in OCM is highly adapted to the considered optimization problem. Consequently, general-purpose nonlinear optimization problem solvers are considered a viable approach to access the design task. An optimization routine based on Sequential Quadratic Programming (SQP) was successfully utilized for topology optimization in [9] and inspired application of this method to the considered problem. Present study lays the foundation for future fundamental research on structural optimization where it is desired to have the possibility to intervene in the structural optimization process. For this reason, a customized topology optimization routine was built and implemented in Matlab. This paper is organized as follows: The regarded optimization problem as well as its characteristics with respect to consideration of self-weight are presented. The implemented topology optimization routine and its components are explained. Finally, the optimization results for pure self-weight load and a combination of design-independent external and self-weight load are presented and discussed.

2 | OPTIMIZATION PROBLEM

In this two-parted section, the topology optimization problem considering self-weight is formulated, its characteristics as well as measures necessary to enable its solution are presented. Subject of the present study is optimization of structural stiffness with a bound on disposable amount of material. The optimization problem is formulated on a design space discretized by finite elements which enables access to the optimization problem and structural analysis.

2.1 | Formulation

The optimization task in the discretized design space is given in terms of a material distribution problem. It is determined for each finite element whether it has to be occupied by material to carry a certain load. The material distribution is expressed by the design variables χ. Void elements are represented by $\chi_e = \chi_{\min} \approx 0$ and elements that are occupied by material by $\chi_e = 1$. A suitable measure for static structural stiffness is the elastic strain energy $W(\chi)$ in its discretized formulation

$$f(\chi) = 2\,W(\chi) = \mathbf{r} \cdot \mathbf{u} = \sum_{e=1}^{N_e} \mathbf{u}_e \cdot \mathbf{k}_e\, \mathbf{u}_e\ . \tag{1}$$

The bound on disposable material is expressed by a maximum volume fraction $\bar{\chi}$ of the design space to be occupied by material

$$g(\chi) = \sum_{e=1}^{N_e} \chi_e - N_e\,\bar{\chi} \leq 0\ . \tag{2}$$

Since the result from structural analysis $\mathbf{u}$ is used to evaluate the objective function $f(\chi)$, it is guaranteed that the material distribution for each iteration fulfills the discretized balance of momentum of the structure $\mathbf{k}\,\mathbf{u} = \mathbf{r}$.

Consideration of self-weight introduces design dependent structural loading in terms of a linear relation between element load vector $\mathbf{r}_e$ and the design variable χ_e corresponding to that element, which impacts the gradient of the objective function with respect to the design variables

$$\frac{\partial f}{\partial \chi_e} = \underbrace{-\mathbf{u}_e \cdot \frac{\partial \mathbf{k}_e}{\partial \chi_e}\,\mathbf{u}_e}_{<0\,\forall\, e} + \underbrace{2\,\frac{\partial \mathbf{r}_e}{\partial \chi_e} \cdot \mathbf{u}_e}_{>0\,\forall\, e}\ . \tag{3}$$

In contrast to pure external loading, the objective function displays a non-monotonous behavior with respect to the design variables, which significantly impacts the optimization results.

2.2 | Relaxation

Due to the high number of design variables and the strong nonlinearity of the optimization problem, its discrete-valued form cannot efficiently be solved. To enable access for methods of mathematical programming, the optimization problem is relaxed by introduction of intermediate material densities $0 \leq \chi_{\min} \leq \chi_e \leq 1$ in the design space. Since a clear "black-and-white" material distribution is desired, intermediate material densities are penalized using the popular

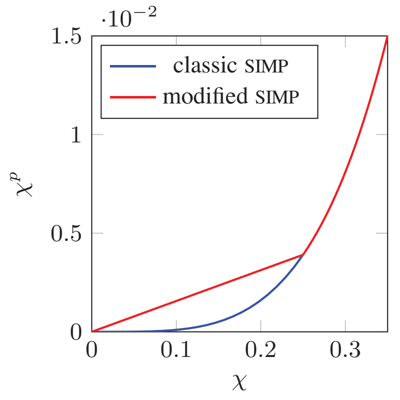

FIGURE 1 Classic versus modified SIMP for $p = 4$.

SIMP-approach (single isotropic material with penalization of intermediate densities) [2]

$$\boldsymbol{C}_e(\chi_e) = [\chi_e]^p\, \boldsymbol{C}^0, \qquad p > 1\,. \tag{4}$$

The idea is to lower the contribution to structural stiffness of elements with intermediate material densities in comparison to their material consumption. When self-weight is considered, the classic SIMP-approach leads to unconstrained deformation of elements with low material density and, in the optimization result, to erratic intermediate material densities. In order to prevent this, the classic SIMP-approach was modified by Bruyneel and Duysinx in [8]

$$\boldsymbol{C}_e(\chi_e) = \begin{cases} \chi_e \left[\chi_c^{p-1}\, \boldsymbol{C}^0\right] & \text{if} \quad \chi_{\min} \leq \chi_e \leq \chi_c \\ & \text{with} \quad \chi_c = 0.25 \\ [\chi_e]^p\, \boldsymbol{C}^0 & \text{if} \quad \chi_e > \chi_c\,, \end{cases} \tag{5}$$

by establishing a linear relation between material density and element stiffness for low material densities. The classic and the modified SIMP-approach are illustrated in Figure 1. Albeit Equation (5) represents a non-differentiable function at χ_c, its application did non lead to any numeric issues in gradient-based optimization. This can be explained by the fact that the sign of the derivative and, as a result, of the search direction, does not change at χ_c. Since χ_c represents a material density with comparably high mass consumption in relation to contribution to stiffness, the algorithm tends to push the material density of elements with $\chi_e = \chi_c$ away from this value. Furthermore, the non-smooth point is not part of the optimization result, and is therefore not critical for evaluation of convergence.

3 | TOPOLOGY OPTIMIZATION USING SQP

In this section, the routine applied to solve the relaxed topology optimization problem is presented. The optimization routine is illustrated and summarized in Algorithm 1. The results of topology optimization exhibit strong mesh dependency in the form of increasingly complex structures with refinement of the FE discretization. To control the minimum structural dimensions and to prevent checkerboarding, the heuristic sensitivity filter by Sigmund, presented in [10], is applied. The filter is based on a convolution operator applied for modification of the element sensitivities. The entries of the convolution operator $\hat{\mathbf{H}}$ consist of the distance between the centers of the neighboring elements within a predefined filter radius r_f. The element sensitivities are modified by

$$\begin{aligned} \hat{\mathbf{H}}_i &= \max[r_f - \mathrm{dist}(e,i),\, 0], \\ \frac{\partial \hat{f}}{\partial \chi_e} &= \frac{1}{\chi_e \sum_{i=1}^{N_e} \hat{\mathbf{H}}_i} \sum_{i=1}^{N_e} \hat{\mathbf{H}}_i\, \chi_i\, \frac{\partial f}{\partial \chi_i}. \end{aligned} \tag{6}$$

ALGORITHM 1 Topology optimization routine.

Data: iteration index $k = 0$
initial values material distribution $\chi_0 = \bar{\chi}$
Lagrange multiplier $\mu_0 = 0.1$
approximated Hessian (BFGS) $\mathbf{B}_0 = \mathbf{I}$
Result: optimum variable set χ^*, μ^*
structural analysis $\mathbf{u}(\chi_0)$
evaluation objective and constraint function $f_0 = f(\chi_0), \quad \nabla_\chi f_0 = \nabla_\chi f(\chi_0)$
$g_0 = g(\chi_0), \quad \nabla_\chi g_0 = \nabla_\chi g(\chi_0)$
sensitivity filter $\nabla_\chi \hat{f}_0$
while $\frac{|f_{k+1} - f_k|}{|f_k|} > 10^{-5}$ **do**
  solution of quadratic approximation $\Delta\chi_k, \Delta\mu_k$
  while $\phi(\alpha) > \phi(0) + c_1\,\alpha\phi'(0)$ **do**
    update primal and dual variables χ_{k+1}, μ_{k+1}
    structural analysis $\mathbf{u}(\chi_{k+1})$
    evaluation of objective and constraint function $f_{k+1}, \nabla_\chi f_{k+1}, g_{k+1}, \nabla_\chi g_{k+1}$
    evaluation merit function $\phi(\alpha)$
  end
  sensitivity filter $\nabla_\chi \hat{f}_{k+1}$
  BFGS-update $\mathbf{B}_{k+1}$
  iteration index $k \leftarrow k + 1$
end

The continuous optimization problem to be solved is inequality constrained and the design variables are subject to box constraints. The objective and constraint function are combined to the Lagrangian $L(\chi, \mu) = f(\chi) + \mu\, g(\chi)$ in terms of which the optimality criteria, the Karush-Kuhn-Tucker-conditions, are given:

$$\begin{aligned} \nabla_\chi L(\chi^*, \mu^*) = \nabla_\chi f(\chi^*) + \mu^* \nabla_\chi g(\chi^*) = \mathbf{0} \\ \mu^*\, g(\chi^*) = 0 \qquad \text{with} \quad \mu^* \geq 0\,. \end{aligned} \tag{7}$$

SQP, an approximation technique for strongly nonlinear optimization problems, established by Schittkowski, see [11], is applied to solve the given optimization problem. This method was chosen due to its high versatility regarding the optimization problem.

The numeric solution scheme is based on a quadratic approximation of the original optimization problem in every iteration step k. Solution of the approximated problem yields the search direction for the design variables $\Delta\chi_k$ and the Lagrange multiplier $\Delta\mu_k$. Since exact evaluation of the Hessian is in many cases computationally expensive, which adversely affects the efficiency of the algorithm, the well-known BFGS-approximation [12] $\mathbf{B} \approx \nabla_{\chi\chi} L(\chi, \mu)$ is applied and complemented by damping [13], to ensure a positive definite approximation. Application of an approximation technique for the Hessian makes a line search procedure necessary in order to determine the step size α to the given search direction. Since the Lagrangian takes a saddle point for the optimum parameter set χ^*, μ^*, a merit function $\phi(\alpha)$ is introduced to evaluate the optimization progress with respect to the step size [14]. The merit function consists of the objective function and an expression penalizing constraint violation. Those terms are evaluated at the parameter set which was updated using a trial step size α

$$\chi_{k+1} = \chi_k + \alpha\,\Delta\chi_k \qquad\qquad \mu_{k+1} = \mu_k + \alpha\,\Delta\mu_k\,. \tag{8}$$

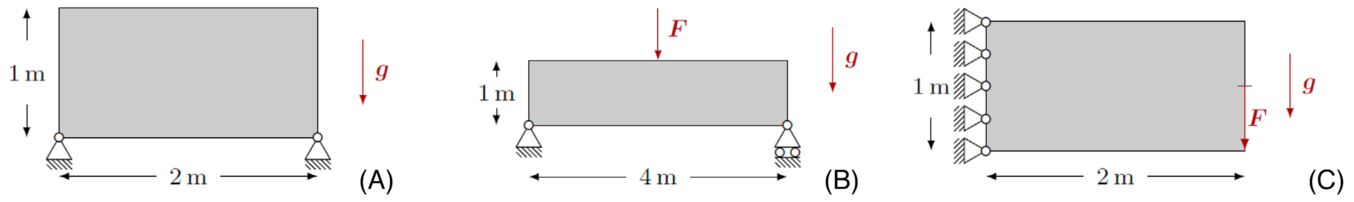

FIGURE 2 Load cases: (A) pure self-weight, (B) MBB-beam, and (C) cantilever beam.

TABLE 1 Material data steel.

	data
ρ [kg m^{-3}]	7850
E [Pa]	$210 \cdot 10^9$
ν [-]	0.3
σ^y [Pa]	$235 \cdot 10^6$

The merit function thus solely depends on the step size. In the present study, the differentiable augmented Lagrange merit function is applied. As presented in [15], it is given by

$$\phi(\alpha) = f(\chi_{k+1}) + \begin{cases} \mu \max[g(\chi_{k+1}), 0] - \frac{1}{2}\, r\,(\max[g(\chi_{k+1}), 0])^2 & \text{if} \quad \max[0, g(\chi_{k+1})] \le \frac{\mu}{r} \\ \frac{\mu^2}{2\, r^2} & \text{else} \end{cases} \tag{9}$$

with r serving as a penalty parameter. The line search is considered successful when the sufficient decrease condition

$$\phi(\alpha) \le \phi(0) + c_1\, \alpha\, \phi'(0) \tag{10}$$

is fulfilled. Structural analysis is carried out for each trial step size which makes an efficient line search procedure a vital component for the overall performance of the optimization routine.

4 | RESULTS and DISCUSSION

In this section, the results from structural optimization are presented. The presented optimization routine was implemented in Matlab. Three plane load cases, characterized by pure self-weight loading and a combination of self-weight, and constant external loading, are regarded and illustrated in Figure 2. Besides pure self-weight loading, the Messerschmitt-Bölkow-Blohm (MBB) beam, a popular benchmark problem for topology optimization [3, 7], as well as a classic cantilever beam are investigated. Focus of evaluation are the resulting structures as well as their structural properties. Structural load is evaluated using the von Mises equivalent stress σ^{VM} which is compared to the yield stress σ^{y} of the underlying material. The design space is discretized using finite elements, linear quadrilateral Lagrange elements with an aspect ratio of 1 are used. Symmetry boundary conditions were applied to the load cases pure self-weight and MBB-beam.

Structural analysis is carried out with element numbers of $N_e = 3600$ for pure self-weight and $N_e = 7200$ for the MBB-beam and the cantilever beam. A material density χ_e is assigned to each element. A plane stress state is assumed in the design space. For the material to be distributed, the parameters of steel, see Table 1, were applied. The optimization parameters are summarized in Table 2.

4.1 | Pure self-weight loading

The result from topology optimization for pure self-weight loading and $\bar{\chi} = 0.5$ is presented in Figure 3. The structure has the shape of an arc. Good accordance with the catenary curve, which is described by the hyperbolic cosine function,

TABLE 2 Optimization data.

	data
Penalty factor p	4
Filter radius r_f	0.1
Convergence criterion	$\frac{\lvert f_{k+1}-f_k\rvert}{f_k} \leq 10^{-5}$

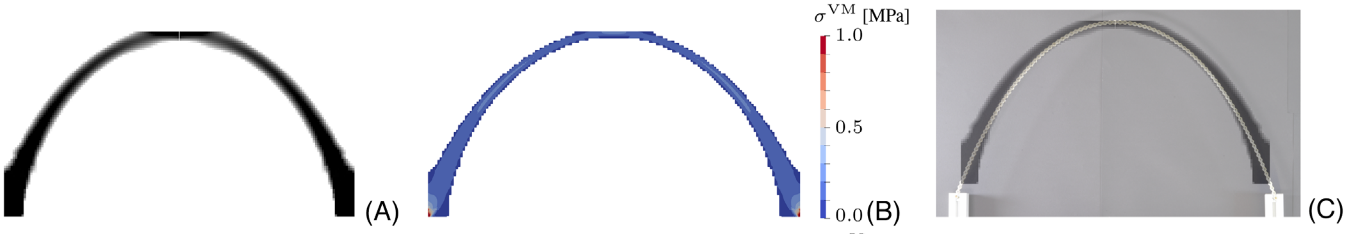

FIGURE 3 Result of pure self-weight loading (A) material distribution, (B) equivalent stress σ^{VM}, and (C) comparison to inverted hanging chain.

TABLE 3 Variation maximum amount of material $\bar{\chi}$.

$\bar{\chi}$	$\bar{\chi}^*$	$f(\chi^*)$ [J]
0.2	0.200	$22.3 \cdot 10^{-3}$
0.4	0.204	$20.3 \cdot 10^{-3}$
0.6	0.160	$19.0 \cdot 10^{-3}$

could be verified. After ≈ 500 iterations the convergence criterion is reached. Utilization of the modified SIMP-approach successfully prevents unconstrained deformation of elements with low material densities and, as a result, erratic intermediate material densities in the resulting structure. The equivalent stress σ^{VM} is homogenously distributed in the resulting structure, the maximum value is given by $\sigma^{VM}_{max} = 1.8$ MPa which is much lower than the yield stress σ^y of the underlying material and proves the load bearing capacity of the structure.

Variation of $\bar{\chi}$ has little impact on the optimum material distribution χ^* or the value of the objective function $f(\chi^*)$. The optimum structure covers volume fractions $\bar{\chi}^*$ between 16% and 20% of the design space with material, see Table 3. Interestingly, no lower bound on the amount of material in the design space must be defined. This outcome is traced back to the non-monotonous behaviour of the objective and complies with the findings described in [8, 16].

The absence of prescription of a minimum amount of material does not result in convergence to the physical optimum, which is given by absence of material in the design space. Convergence to the presented optimum is explained by application of the SIMP model to the optimization problem. Prescription of χ_{min} to each finite element of the design space results in a higher value of the objective function than the arc-structures resulting from optimization.

The optimization result is studied in terms of principal stresses. The first principal stress is defined as the one with the highest absolute value. The first principal stress is, in the vast majority of the structure, a compression stress and about 10 times as large as the second principal stress. The first principal stress as well as its orientation is illustrated in Figure 4. It

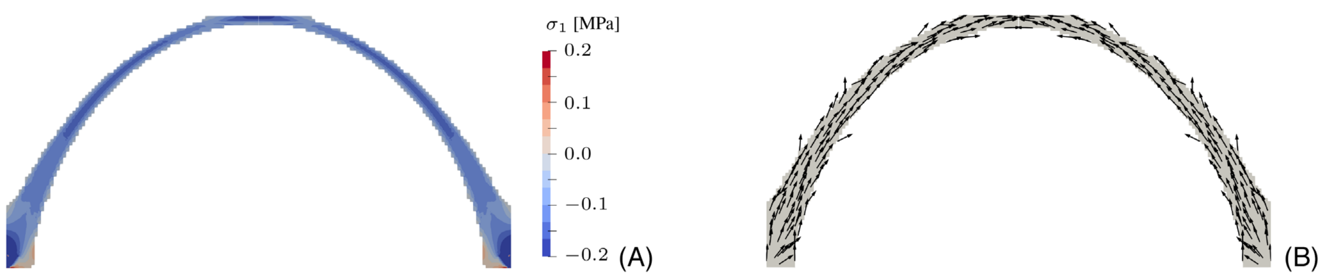

FIGURE 4 Pure self-weight loading: principal stresses (A) distribution and (B) orientation.

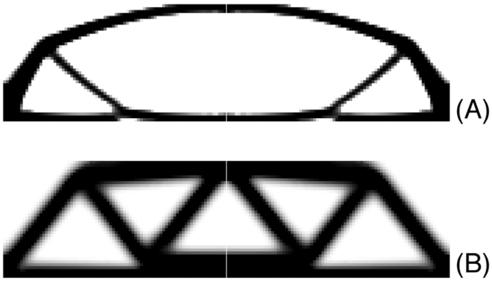

FIGURE 5 Result of MBB beam for pure loading by (A) self-weight and (B) external load.

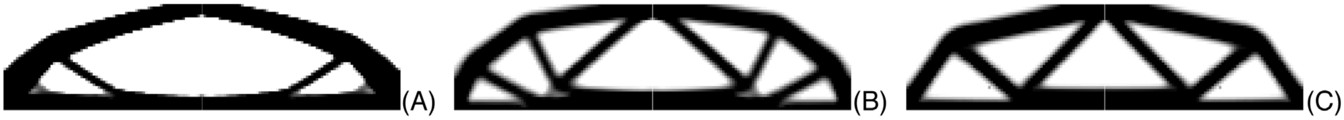

FIGURE 6 Result of MBB beam for combined loading by external force and self-weight with (A) $\frac{F}{F_0^g} = 0.2$ and (B) $\frac{F}{F_0^g} = 1$ and (C) $\frac{F}{F_0^g} = 6$.

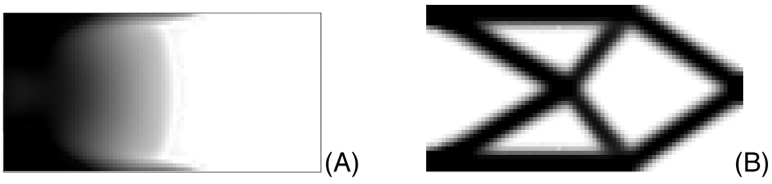

FIGURE 7 Result cantilever beam for pure loading by (A) self-weight and (B) external load.

shows only little variation in magnitude within the majority of the structure and is oriented in direction of the structure, consequently the second principal stress is oriented normal to it. Those findings sustain the observed similarities with the catenary since a hanging chain can exclusively transfer load by normal forces.

4.2 | Mixed load cases

In this section the optimization results for structures undergoing self-weight loading in combination with constant external forces are considered. The ratio between external force F and the absolute value of initial self-weight of the structure F_0^g is varied in order to study its impact on the optimization result. For reference, the results for the extreme load cases of pure self-weight loading and exclusive loading by an external force of $F = 10^5$N are given.

The MBB beam is considered first. The reference results are illustrated in Figure 5. For pure self-weight loading, the volume fraction of material in the resulting distribution is with $\bar{\chi}^* = 0.26$ considerably lower than the prescribed maximum $\bar{\chi} = 0.5$, and the characteristic arc shape is observed. In contrast, the result for pure constant external load $F = 10^5$N occupies a fraction of $\bar{\chi}^* = \bar{\chi}$ of the design space with material which was observed for all load cases with design independent load. The results from the MBB beam undergoing a combination of external and self-weight load are illustrated in Figure 6. It is apparent that the resulting structures display more similarities with the reference load case which dominates in the mixed load.

Nevertheless, even when external loading is strongly dominant, the impact of self-weight is clearly visible in the optimization result. The results were compared to the ones presented in [8, 17] and satisfactory accordance is observed although the optimizers in the referenced publications differ from the one applied in the present study as well as each other.

An analogous study was carried out for a cantilever beam, the reference results are presented in Figure 7. Although the result for pure self-weight loading does not seem meaningful in terms of structural design, it can still be interpreted as an indication for possibilities to improve the layout of structures that experience a load profile inducing a strong bending

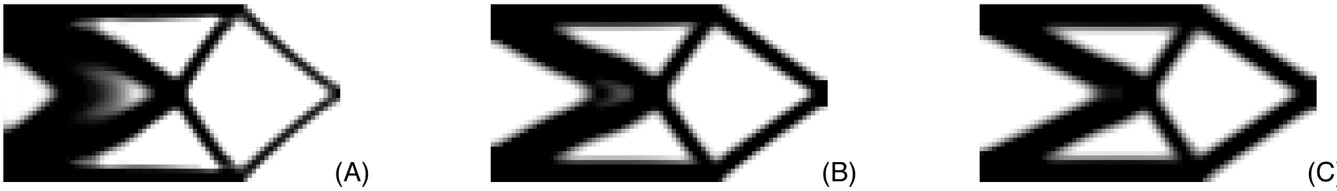

FIGURE 8 Result of cantilever beam for combined loading by external force and self-weight with (A) $\frac{F}{F_0^g} = 0.2$ and (B) $\frac{F}{F_0^g} = 1$ and (C) $\frac{F}{F_0^g} = 6$.

moment: the material is transferred by the optimizer towards the support which reduces the bending moment in the structure. Since the right edge of the design space is load-free, there is no reason to gather material in this location. The result for pure external loading is characterized by homogeneous structural dimension and is optically identical with results that can be found in literature, see [7].

Variation of the ratio of external and self-weight loading results in the structures illustrated in Figure 8. Similarly to the MBB beam, it is observed that, depending on which load is dominant, the resulting structure has more similarities with the correspondent reference result. Even for clear dominance of external loading, it is observed that the dimension of the structural components nearby the support is larger than at the point of force application. The tendency of the optimizer to minimize the bending moment in the resulting structure is therefore still observable. Similar results were obtained in [7].

5 | CONCLUSION AND OUTLOOK

In this study topology optimization considering self-weight was carried out using a SQP-based routine. The material interpolation model was modified to account for unconstrained deformation of elements with low material densities. It was found that the modified SIMP-approach delivers satisfactory results. The simulation results were compared to literature and validate the functionality of the implemented optimization routine. Pure self-weight loading as well as mixed load cases were considered. The shape of the resulting structure for pure self-weight loading shows good compliance with the catenary. The dominant structural load is compression stress oriented in direction of the resulting structure which represents another similarity to the catenary. Albeit the results from topology optimization generally exhibit strong dependency from the initial material distribution, variation of the initial amount of material did not visibly impact the result. By analysis of the resulting structures for all considered load cases, it was found that the equivalent stress was homogenously distributed which represents an advantageous load distribution. Consideration of mixed load cases displays an impact of self-weight on the optimization result even when structural load is significantly dominated by design independent external load. Since structures in civil engineering are preponderantly loaded by their self-weight, the present study highlights the vital character of its consideration in the context of structural optimization in this domain.

It is planned to elaborate the method of topology optimization considering self-weight in future studies. Efforts to increase efficiency of the optimization routine in terms of convergence speed are made. Consideration of anisotropic materials with local optimization of material orientation is envisaged as well as implementation of three dimensional optimization routines.

ACKNOWLEDGMENTS

Open access funding enabled and organized by Projekt DEAL.

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How to cite this article: Masarczyk, D., & Kuhl, D. (2023). Topology optimization considering self-weight. *Proceedings in Applied Mathematics and Mechanics*, *23,* e202300258. https://doi.org/10.1002/pamm.202300258