TWO-MUONIC ATOMS

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X-ray transition energies for two-muonic atoms are calculated. The basis are relativistic self-consistent-field calculations including the corrections normally known in muonic atoms plus the vacuum polarization, magnetic interaction and retardation in the μ-μ-interaction, the specific mass correction and the configuration interaction.

During the last few years accelerators have been constructed (at SIN and in Los Alamos) which will increase the number of stopped muons by several orders of magnitude. Thus it may be possible to look for such exotic systems like two-muonic atoms. The observation of X-rays from such systems is very important because they would provide first direct experimental evidence of the muon-muon interaction.

To the best of our present knowledge the muon-muon interaction can be assumed to be electromagnetic only with no admixture of strong or weak interaction within the present error limits, although several authors [e.g. 1, 2] speculate about a coupling to spin zero mesons or an anomalous interaction of the muon.

The basis for the theoretical description of such two-muonic systems is given by selfconsistent relativistic Dirac-Fock calculations. In addition to the relatively well known corrections already present in normal muonic atoms, like the effect of the extended nucleus, the vacuum polarization and fluctuation correction in the muon-nucleus interaction, the nuclear polarization and the electron screening [3], we have to correct for magnetic and retardation interaction as well as for vacuum polarization in the muon-muon interaction. Furthermore we have to consider configuration interaction, the specific mass correction as well as the additional coupling of some of these effects via the selfconsistent field procedure.

The gross-structure of the two-muonic level diagram in fig. 1 (we are going to discuss the system

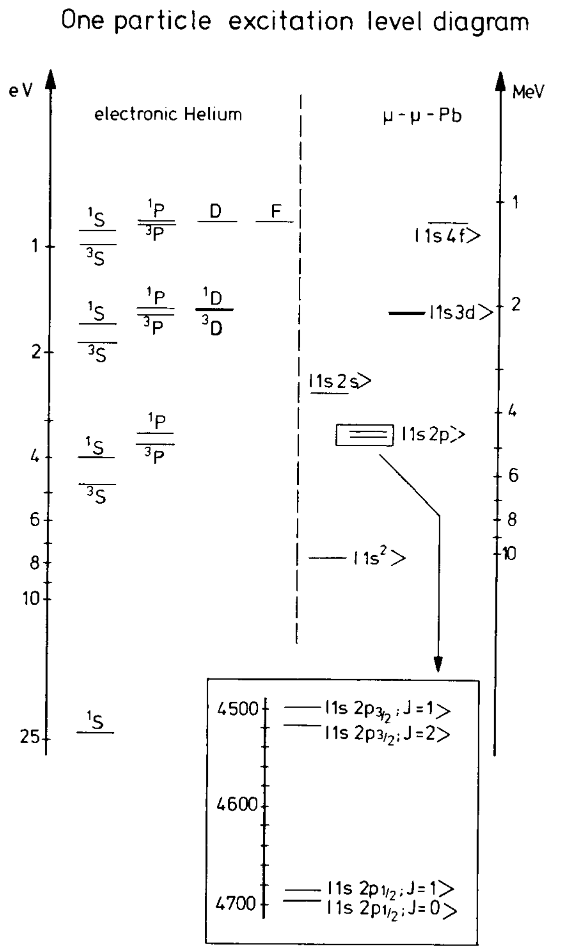

Fig. 1.

μ-μ-Pb with one of the muons in the 1s state) in the first place is similar to the well known one particle excitation level diagram of electronic helium. Both diagrams in fig. 1 are scaled in such a way that the levels with large main quantum numbers and large angular momenta are equally apart. The comparison of both diagrams shows two main differences. First, all $|1s\, ns\rangle$ levels in the heavy two-muonic Pb atom are shifted strongly towards a smaller binding energy compared to the ^{1}S and ^{3}S levels in electronic helium. This effect is due to the monopole interaction of the muon wavefunctions with the extended nucleus and thus is most important for all $|1s\, ns\rangle$ levels having the greatest overlap with the nucleus. The second difference is the relative magnitude of the spin-orbit splitting via the energetic splitting of the coupling to good total angular momentum. In two-muonic lead the spin-orbit splitting is the much larger effect whereas in electronic helium the splitting into the ortho- and para-levels is by far the overwhelming effect.

The second strongest effect in every muonic system (after the influence of the extended nucleus) is the vacuum polarization. This correction up to the first order in $Z\alpha$ plus the Coulomb interaction is given by the expression [4]

$$V(r) = V_{\mathrm{c}} + V_{\mathrm{VP}} = -e^2 \int \frac{\rho(r')}{|r-r'|} \left\{ 1 + \frac{2\alpha}{3\pi} \times \int_1^\infty \exp\left(-\frac{2}{\lambda_e}|r-r'|\xi\right)\left(1+\frac{1}{2\xi^2}\right)\frac{(\xi^2-1)^{1/2}}{\xi^2}\,\mathrm{d}\xi \right\} \mathrm{d}\tau'$$

with $\lambda_e = 386$ fm; $\rho(r)$ is the charge distribution of the source which generates the field, usually the extended nucleus but in the case of the muon-muon interaction it is the charge distribution of the muon. This vacuum polarization interaction has to be introduced four times in the treatment of two-muonic atoms. First it changes the muon-nucleus interaction. In addition the muon-muon interaction has to be corrected for the VP. In a perturbation calculation this leads to two contributions, the direct matrix elements and the exchange matrix elements. The fourth contribution comes from the redistribution of the whole system when the muon-nucleus vacuum polarization is introduced in the selfconsistent field procedure itself.

Then the muon-muon interaction has to be corrected for the magnetic and retardation interaction which usually is given by the Breit Hamiltonian [5]

$$H_{\mathrm{B}} = -\tfrac{1}{2}\left(\frac{\alpha_i\alpha_i}{r_{ij}} + \frac{\alpha_i r_{ij}\alpha_i r_{ij}}{|r_{ij}|^3}\right).$$

The matrix elements appearing in a rigorous relativistic treatment were studied by Grant [6] and Cooper [7]. Three out of the four resulting matrix elements are non zero and lead in our case to a nonnegligible contribution.

Configuration interaction is important only in the case when there are two levels with the same angular momentum and parity and relatively close in energy. Within the lowest levels of our system μ-μ-Pb this effect is only important for the configuration $|1s\, 2p_{1/2}; J=1\rangle$ and $|1s\, 2p_{3/2}; J=1\rangle$, where the first level is lowered by about 1 keV compared to a single configuration calculation.

The last contribution which has to be discussed is the specific mass correction [8]. It appears in addition to the trivial reduced mass correction when the Schrödinger equation of a many body system is separated into the center of mass and relative coordinates. Although no relativistic description has been formulated yet non-relativistic calculations show that its influence is small [9] and in every case less than 0.1 keV.

If we sum up all corrections we obtain the total binding energy of the two-muon system. Because the nuclear polarization is known only within several keV it is not too conclusive to compare these values with the binding energies of the normal muonic system.

Because in the experiment one measures only differences between the energy levels we have listed in table 1 (as an example) those corrections which shift the transition energy $|1s\, 2p_{1/2}; J=1\rangle \to |1s^2\rangle$ relative to the analog transition energy $2p_{1/2} \to 1s$ of the one-muonic atom by more than 0.1 keV. The resulting transition energy lies somewhere between the normal muonic lines of the elements lead ($Z=82$) and thalium ($Z-1$) which are also given because the muons shield themselves by about 0.5 to 1 charge depending on the configuration.

We did not discuss the hyperfine structure splitting due to the coupling of the muons with the nucleus [10] nor with the electrons which are present. But we

Table 1
The corrections for the $|1s\,2p_{1/2}; J=1\rangle \to |1s^2\rangle$ transition in μ-μ-Pb relative to the 2p → 1s transitions in lead and thalium.

		$|1s\,2p_{1/2}; J=1\rangle \to$ $|1s^2\rangle$ in Pb (keV)		
Energy eigenvalue (including in a selfconsistent way the effect of the extended nucleus, reduced mass and μ-N vacuum polarization)		−5690.67		
Vacuum polarization in the μ-μ interaction		−0.33		
Magnetic contribution		+2.55		
Retardation		−0.20		
Configuration interaction		+0.99		
Specific mass correction		−0.02		
Pb 2p → 1s	−5750.35	sum −5687.68	−5677.73	Tl 2p → 1s
Difference	62.67 keV		9.95 keV	

can eliminate the first effect by using spherical nuclei with spin zero and a very large excitation energy such as ^{208}Pb. The coupling to the remaining electrons [11] is only of the order of eV.

We estimate the accuracy of the resulting energetic differences of the analog transition energies in one- and two-muonic atoms to be less than 20 eV, because similar ab initio calculations of binding energies in the much more complicated system of electronic fermium ($Z = 100$) have led to an excellent agreement [12] with the experimental results. The question of feasibility of such an experiment is very difficult because in a realistic estimation one is still many orders of magnitude away from realization. The only hope one might have would be an experiment with a very low Z element as the target and a very high Z element as an impurity in it. This possibly might lead to an enrichment of muons at the location of the high Z element atoms and thus might give rise to X-ray transitions in two-muonic atoms.

As a conclusion we may say that it is possible to determine the muon-muon interaction directly by applying the corrections discussed here if it is possible to measure X-ray lines from two-muonic systems. Thus the main question if the muon behaves like a heavy electron or not could be answered.

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