INTERNATIONAL JOURNAL OF QUANTUM CHEMISTRY, NO. 6, 43–46 (1972)

What Can One Learn with Muons About Atomic Physics?*

B. FRICKE
Department of Materials Science, Northwestern University, Evanston, Illinois 60201

The atomic physics of a hydrogen-like atom is still an interesting field. With this simplest of all possible atomic systems, one gets (besides many other quantities) information about the fundamental question: What is the "true" interaction between charged particles? Although these atoms are relatively simple, one has not been able to study experimentally more than a few very light ones, namely up to C^{+5}. Lamb shift experiments obtain very precise energy differences between $ns_{1/2}$ and $np_{1/2}$ electronic states, which should be degenerate according to the Dirac equation. The contributions to the best known splitting which is the $2s - 2p_{1/2}$ difference in hydrogen are given in column 1 of Table I. After more than 20 years,

TABLE I. The corrections to the binding energies or atomic transitions for $H(2s - 2p_{1/2})$, μ-Pb(1s) and μ-Pb($5g_{7/2}$–$4f_{5/2}$).

	H ($2s_{1/2}$ - $2p_{1/2}$) in MHz	μ - Pb ($1s_{1/2}$) in keV	μ - Pb ($5g_{7/2}$ - $4f_{5/2}$) in KeV
energyeigenvalue or transition energy Dirac equation point nucleus	0.00	-20991.43	-435.90
finite size effect	+0.13	+10457.33	+0.01
2nd order self-energy	-1079.32	+2.30	+0.01
2nd order Vacuumpolarisation	-27.13	-67.15	-2.19
2nd order remainder	+6.76	-	-
4th order self-energy	-0.10	0.00	0.00
4th order Vacuumpolarisation	-0.24	-1.60	-0.04
reduced mass effect	-1.46	+5.75	+0.24
recoil	+0.36	+0.06	0.00
nuclear polarisation	-	-6.	-0.01
electron shielding	-	+0.01	+0.08
difference theory - experiment	+0.05(8)		+0.14(2)

*Work supported by the U.S. Atomic Energy Commission.

theory and experiment agree to within 0.05(8) MHz, but still a degree of agreement which satisfies everyone has not been attained [1]. But the main contributions, vacuum fluctuation or self-energy and vacuum polarization, were established to exist in nature, a number of years ago.

Besides the normal electronic atoms which are the only kind of atoms observed in nature, it is possible to produce and to study atoms where one electron is replaced by a muon, a pion or another elementary particle. But only the muonic atoms are exact analogs of the electronic atoms because the interaction of both the muon and the electron is the same as far as one knows. The only difference is that the mass of the muon is 206 times the electronic rest mass. Thus the binding energies are larger and the orbital radii are smaller by this factor 206, when compared with the analogous hydrogenic electronic atom in the same state. With these muonic atoms, one is able to study hydrogenic atomic orbitals with all possible nuclei. The second advantage is that the probe particle, the muon, is so near to the nucleus, that all effects which depend on the distance from the nucleus become much more significant than in electronic atoms. In column 2 of Table I, one finds the contributions to the $1s$ binding energy in muonic lead [2]. The $1s$ radius is of the order of only 7 fermi which means that the muon is already more than 50% inside the nucleus. Therefore the contributions from the extended nucleus, which permit experimentalists to make a precise determination of the protonic charge distribution, reduces the binding energy for a point nucleus by about one half.

Also, for heavy nuclei the vacuum polarization contribution becomes very large whereas the self-energy becomes very small, even though the latter is dominant in electronic atoms. The vacuum polarization contribution is due to the virtual creation and annihilation of electron-positron pairs which are then polarized in the presence of the field of the nucleus. This polarization effect is strong only in the immediate vicinity of the nucleus and therefore the polarization charge density is mainly inside or very near to a radius which is equal to λ_e the Compton wavelength of the electron. The latter is of the order of 380 fermi. The total net charge which is separated by this field effect is on the order of $\frac{1}{2}$% of the charge of the nucleus. This leads to a deviation from the usually used Coulomb potential. The potential (correct up to the first order) between two charged particles is proportional to $1/r[1 + (2\alpha/3\pi) f(r, \lambda_e)]$ where $f(r, \lambda_e)$ in this expression is a complicated function of r and λ_e, and $\alpha = 1/137$.

This polarization charge and polarization charge density can be most effectively studied in higher muonic transitions, where all the nuclear contributions are very small, but where the orbital radii are still $\lesssim \lambda_e$. We present as an example the transition $5g_{7/2} - 4f_{5/2}$ in muonic lead [3] in column 3 of Table I. The accuracy of this measurement is so good that a small discrepancy between theory and experiment has been found. This suggests that some higher order contributions are either neglected or have not been calculated accurately enough. A possible explanation in terms of nonlinear electrodynamical effects has been proposed [4] but this seems to be very unlikely. A large discrepancy between theory and experiment should be observed especially for lower transitions, but the agreement is always observed to be relatively good. Besides these small differences, the studies of muonic atoms have lead to a much better knowledge of the fundamental problem, the interaction between two charged particles.

Another very interesting contribution to atomic physics is the so-called electron shielding effect. A nucleus with a charge Z does not only bind a muon but also $Z - 1$ electrons at one time. But because the muon is so very near to the nucleus, the outer electrons see an inner charge which is only about $Z - 1$. On the other hand, the fractional probability of finding electrons inside the muonic orbitals changes when the muon is cascading down from the outer shells where the muon is captured. This leads to different shielding contributions to the binding energy of the muon in various levels due to the presence of these electrons. For some transitions, this can lead to corrections as high as 1 keV or more. Muonic (γ)-ray spectra can be used as an experimental tool to determine the electronic charge distribution inside a radius of about 0.3 Å These values can be compared with self-consistent field calculations which take into account the muon plus the electrons such as we have done some time ago [5].

Going back to electronic atoms, with the present greater knowledge of all contributions which arise and with the self-consistent Dirac-Fock calculations, one is able to determine more accurate binding energies for the inner electrons of heavy atoms. In Table II we present the contributions for the inner electrons for the very high Z element, fermium [6]. In these calculations only the more precise interaction between the nucleus and the electrons (in the vacuum polarization contribution) has been taken into account. In principle, the interaction

TABLE II. Theoretical and experimental Dirac-Fock inner electron binding energies for fermium plus all known corrections.

Level	1s	2s	$2p_{1/2}$	$2p_{3/2}$	3s	$3p_{1/2}$
Electric energy	-142.929	-27.734	-26.791	-20.947	-7.250	-6.815
Magnetic energy	+0.715	+0.091	+0.153	+0.092	+0.019	+0.033
Retardation[a]	-0.041	-0.008	-0.013	-0.011	-0.001	-0.003
Vacuumfluctuation	+0.457[b]	+0.096	+0.009	-0.003	+0.025	+0.003
Vacuumpolarisation	-0.155	-0.026	-0.004	+0.000	-0.006	-0.001
Total energy	-141.953(26)	-27.581(20)	-26.646(10)	-20.869(10)	-7.213(15)	-6.783(4)
Experimental value[c]	-141.963(13)	-27.573(8)	-26.644(7)	-20.868(7)	-7.200(9)	-6.779(7)

[a] Extrapolation taken from J. B. Mann and W. R. Johnson, Phys. Rev. **A4**, 41 (1971).
[b] Extrapolation taken from A. M. Desiderio and W. R. Johnson, Phys. Rev. **A3**, 1267 (1971).
[c] F. T. Porter and M. S. Freedman, Phys. Rev. Letters **27**, 293 (1971).

between the electrons themselves has also to be modified slightly. An estimation for this contribution to the 1*s* binding energy in fermium is of the order of 10 eV, which is near the limit of the present experimental precision, but is not negligible.

As a conclusion, one may say that it has been possible to study the "true" interaction between two charged particles more precisely with muonic atoms than with usual electronic atoms. This knowledge, together with Dirac-Fock calculations, enables one to calculate the precise binding energies of electrons for very heavy Z elements, which leads to a prediction of X-ray spectra. This is especially important because the precise determination of X-ray energies appears to be one of the few ways to identify superheavy elements.

Bibliography

[1] Experimental and theoretical references can be found in: T. Appelquist and J. S. Brodsky, Phys. Rev. Letters **24**, 562 (1970).
[2] H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, R. D. Barton, and D. Kessler, Phys. Rev. **187**, 1565 (1969).
[3] M. S. Dixit, H. L. Anderson, C. K. Hargrove, R. J. McKee, D. Kessler, H. Mes, and A. C. Thompson, Phys. Rev. Letters **27**, 878 (1971).
[4] J. Refelski, L. P. Fulcher, and W. Greiner, Phys. Rev. Letters **27**, 958 (1971).
[5] B. Fricke, Nuovo Cim. Letters **2**, 859 (1969).
[6] B. Fricke, J. P. Desclaux, and J. T. Waber, Phys. Rev. Letters **28**, 714 (1972).