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Detection of notches and cracks based on the monitoring of local strain and the solution of inverse problems

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Abstract
Engineering structures are in general exposed to cyclic or stochastic mechanical loading. Exhibiting incipient cracks, particularly light-weight shell and plate structures, suffer from fatigue crack growth, limiting the life time of the structure and supplying the risk of a fatal failure. Due to the uncertainty of loading boundary conditions and the geometrical complexity of many engineering structures, numerical predictions of fatigue crack growth rates and residual strength are not reliable. Most experimental monitoring techniques, nowadays, are based on the principle of wave scattering at the free surfaces of cracks. Many of them are working well, supplying information about the position of cracks. One disadvantage is that those methods do not provide any information on the loading of the crack tip. In this work, the development of a concept for the detection of straight and simply kinked notches or cracks in finite plate structures under mixed mode loading conditions is presented. In this approach, the distributed dislocation technique is applied to model the direct problem, and a genetic algorithm is used to solve the inverse problem. Solving the inverse problem, eg, with a genetic algorithm, this allows the identification of external loading, crack or notch position parameters, such as length, location or angles, and the calculation of stress intensity factors, as long as the shapes and the number of the cracks are a priori known. Experiments are performed using plates with notches under tensile loading.

KEYWORDS
distributed dislocations, inverse problem, kinked crack detection, notch detection

1 | INTRODUCTION

The dislocation method has mostly been restricted to calculate the stress intensity factor (SIF) in infinite and semi-infinite plate structures. A few works on modelling finite bodies by continuous distributions of dislocations are reported. Sheng[1] has combined the boundary element method with the dislocation technique. Dai[2] has modelled cracks in finite bodies by distributed dislocation dipoles. Han and Dhanasekar[3] have modelled cracks in arbitrarily shaped finite bodies by distributions of dislocations using complex functions. Zhang et al[4] have calculated the elastic fields of a finite plate containing a circular inclusion by the distributed dislocation method.

As an approach, different from the finite element method (FEM), cracks are modelled by a collocation of discrete dislocations. Within a continuum mechanics framework, these dislocations are no lattice defects but displacement discontinuities describing the local crack opening displacement. Thus, it is not necessary to discretize the domain around the crack, considerably saving computation time and data, which is crucial for an efficient solution of the inverse problem. The power of this method further lies in the efficiency to accurately model the singularity at the crack tip. Another advantage is that the solutions for the stress field created by a dislocation are available in a closed form for a wide range of geometries.[5] The solution for a dislocation in a half-plane can be found by a suitable choice of elastic constants from the solution for two bonded half-planes.[6]

The goal of our work is the development of a monitoring concept supplying both the information on the actual crack position and length and the SIF in a plate structure during operation of a system. This enables a more comprehensive and reliable survey of structures, based on both the knowledge of the actual crack position and a numerical prediction of further crack development from crack tip loading parameters. The concept, however, requires the a priori knowledge of crack numbers and shapes. Straight cracks are favourable for the investigations, coming along with the least number of unknowns to identify. Curved cracks can be approximated by a polygonal arrangement of shorter straight cracks, whereupon the most simple case is addressed here in terms of a simply kinked crack.

In Bäcker,[7] a related goal is pursued interpreting electric signals from a polymeric piezoelectric foil attached to the surface of the structure. There, the crack tip near field is used for crack parameter identification. Maheshwari et al[8] have investigated a health monitoring of structures using multiple smart materials. In Boukellif and Ricoeur,[9] a sensor concept was realized numerically and experimentally, applying the body force method to infinite and semi-infinite plate structures with single cracks and exploiting strain data far from the crack. The inverse problem was solved applying the particle swarm optimization (PSO) algorithm. The number of unknowns to be determined, however, was comparably small, unless restricting to the simple case of a Griffith crack. There are also several works on crack detection using the XFEM to solve the direct problem and, eg, the genetic algorithm for solving the inverse problem in the sense of a parameter optimization.[10-13] In Gadala and McCullough,[14] the solution of the direct problem is realized by using the FEM. The method of proper orthogonal decomposition (POD) has also been used to solve inverse crack problems.[15-17] In all these works, cracks are detected, but information about SIF and external loads are not provided. Furthermore, the application of spatial discretisation schemes for solving the crack problems is expensive from the computational point of view and is not very flexible due to sophisticated requirements of crack tip meshing.

2 | THEORETICAL BACKGROUND

The dislocation method is a current approach to determine the SIF for plane cracks under arbitrary load. In this method, the cracks are modeled as distributed dislocation densities along the line of the crack.

The stresses at a field point (x,y) in an elastic plane, induced by an infinitesimal single dislocation with components b_x and b_y of the Burgers vector located at the source point (ξ,η), can be written in global coordinates as

$$\begin{bmatrix} \overline{\sigma}_{xx}^{D}(x,\, y) \\ \overline{\sigma}_{yy}^{D}(x,\, y) \\ \overline{\sigma}_{xy}^{D}(x,\, y) \end{bmatrix} = \frac{2\mu}{\pi(\kappa+1)} \begin{bmatrix} G_{xxx}(x,\, y;\, \xi,\, \eta) & G_{yxx}(x,\, y;\, \xi,\, \eta) \\ G_{xyy}(x,\, y;\, \xi,\, \eta) & G_{yyy}(x,\, y;\, \xi,\, \eta) \\ G_{xxy}(x,\, y;\, \xi,\, \eta) & G_{yxy}(x,\, y;\, \xi,\, \eta) \end{bmatrix} \begin{bmatrix} b_x(\xi,\eta) \\ b_y(\xi,\eta) \end{bmatrix}, \tag{1}$$

where the Kolosov's constant κ is related to Poisson's ratio ν as $\kappa = (3-\nu)/(1+\nu)$ for plane stress and $\kappa = (3-4\nu)$ for plane stain and μ is the shear modulus. The dislocation influence functions G_{ijk}[5,18] describe stresses at a field point (x,y) with a unit Burgers vector acting at (ξ,η). The first index $i = x,y$ in the influence functions indicates the direction of dislocations, whereas the second and third $jk = xx,yy,xy$ denote the components of induced stresses. In general, the influence function can be split into two parts as follows:

$$G_{ijk}(x,\, y;\, \xi,\, \eta) = G_{ijk}^{s}(x,\, y;\, \xi,\, \eta) + G_{ijk}^{r}(x,\, y;\, \xi,\, \eta), \tag{2}$$

where $G_{ijk}^{s}(x,\, y;\, \xi,\, \eta)$ denote the singular part or Green's function, containing the Cauchy kernel in the integral equation. These functions are used to calculate the induced stresses in an infinite medium, see Figure 1.

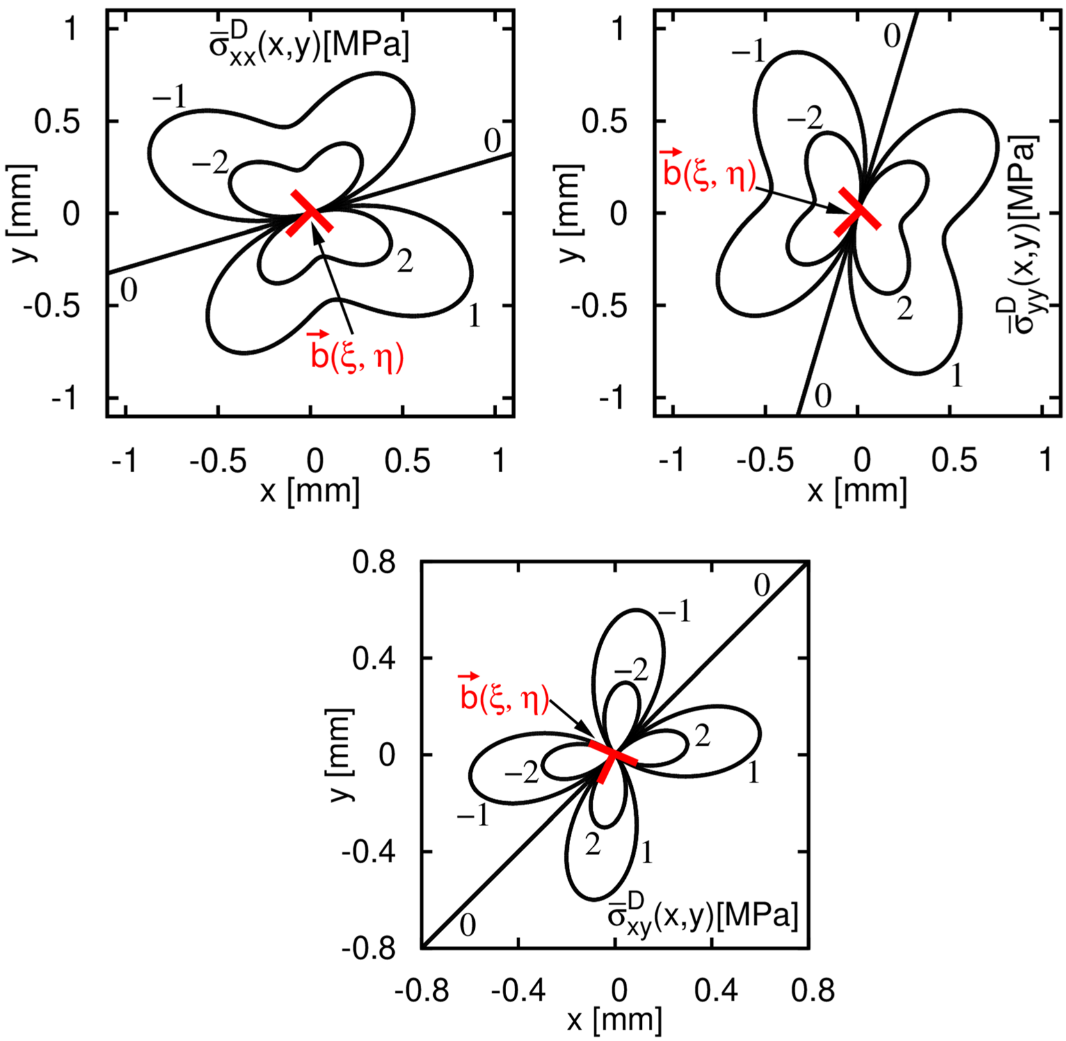

FIGURE 1 Contours of arising stress fields $\overline{\sigma}_{ij}^{D}(x, y)$, $ij = (xx, yy, xy)$ due to an infinitesimal single dislocation with coordinates $b_x = b_y = 10^{-4}$mm located at $(\xi, \eta) = (0{,}0)$ and assuming plane stress conditions with Young's modulus E = 72 000 MPa and Poisson's ratio $\nu = 0.3$

The part $G^{r}_{ijk}(x, y; \xi, \eta)$ denotes the regular functions, accounting for any boundaries or free surfaces. The relationship between the infinitesimal dislocation and the dislocation density $B\left(\widehat{\xi}\right)$ is defined as

$$d\overrightarrow{\widehat{b}} = \left(db_{\widehat{x}}\, db_{\widehat{y}}\right) = \begin{pmatrix} B_{\widehat{x}}\left(\widehat{\xi}\right)d\widehat{\xi} \\ B_{\widehat{y}}\left(\widehat{\xi}\right)d\widehat{\xi} \end{pmatrix}, \tag{3}$$

where $B\left(\widehat{\xi}\right)d\widehat{\xi}$ represents the number of dislocations in the interval $\left[\widehat{\xi}, \widehat{\xi} + d\widehat{\xi}\right]$.

Considering Equation (1), replacing the dislocations b_x and b_y by db_x and db_y and accounting for Equation (3), the stresses induced by continuously distributed dislocations along the crack line in local coordinates $(\widehat{x}, \widehat{y})$ are calculated as follows:

$$\begin{bmatrix} \sigma_{\widehat{x}\widehat{x}}^{D}(\widehat{x},\widehat{y}) \\ \sigma_{\widehat{y}\widehat{y}}^{D}(\widehat{x},\widehat{y}) \\ \sigma_{\widehat{x}\widehat{y}}^{D}(\widehat{x},\widehat{y}) \end{bmatrix} = \frac{2\mu}{\pi(\kappa+1)} \int_{-a}^{a} \begin{bmatrix} G_{\widehat{x}\widehat{x}\widehat{x}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) & G_{\widehat{y}\widehat{x}\widehat{x}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) \\ G_{\widehat{x}\widehat{y}\widehat{y}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) & G_{\widehat{y}\widehat{y}\widehat{y}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) \\ G_{\widehat{x}\widehat{x}\widehat{y}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) & G_{\widehat{y}\widehat{x}\widehat{y}}\left(\widehat{x},\widehat{y};\widehat{\xi}\right) \end{bmatrix} \begin{bmatrix} B_{\widehat{x}}\left(\widehat{\xi}\right) \\ B_{\widehat{y}}\left(\widehat{\xi}\right) \end{bmatrix} d\widehat{\xi}. \tag{4}$$

Equation (4) gives a set of singular integral equations with Cauchy kernels, which can be solved using Gauss-Chebyshev numerical quadrature. The dislocation densities $B_k\left(\hat{\xi}\right)$ are determined accounting for boundary conditions. The first condition is that the crack surfaces are traction free. Secondly, the stresses on the external boundaries are equal to the subjected boundary loads. Finally, the displacement jumps at the crack tips are equal to zero, and the gradient fields at this points are singular. Once having computed $B_k\left(\hat{\xi}\right)$, the strain at arbitrary points is calculated assuming plane stress conditions.

A rectangular plate is introduced as a cut-out from an infinite elastic domain, with dislocations distributed along the intended boundary. The additional equation for the corners is that the values of the dislocation densities are equal for both edges involved.

3 | KINKED CRACK AND NOTCH DETECTION AND PARAMETER IDENTIFICATION

3.1 | Numerical verifications

First verifications have been carried out numerically. The strain $\varepsilon_{ij}(P_m)$ is "measured" at points P_m representing the positions of virtual strain gauges aligned along the edges of a rectangle with corner coordinates $(\overline{x}, \overline{y})$ and $(\widetilde{x}, \widetilde{y})$.

The first example is a finite plate (30 mm × 30 mm) with kinked crack and the strain $\varepsilon_{ij}(P_m)$ emerging from the distributed dislocation technique at $P_m(m = 1, ..., 12)$ measuring points, $\left(\overline{x}_{(a)}; \overline{y}_{(a)}\right) = (1;\ 29)$ mm, $\left(\widetilde{x}_{(a)}; \widetilde{y}_{(a)}\right) = (29;\ 1)$ mm, as shown in Figure 2 (left). The second example is a finite plate (200 mm × 200 mm) with two notches. The strain $\varepsilon_{ij}(P_m)$ of the direct problem emerges from the FEM at $P_m(m = 1, ..., 12)$ measuring points, $\left(\overline{x}_{(b)}; \overline{y}_{(b)}\right) = (25;\ 154)$ mm, $\left(\widetilde{x}_{(b)}; \widetilde{y}_{(b)}\right) = (175;\ 46)$ mm, as shown in Figure 2 (right). The inverse problem in both cases is solved based on the dislocation technique.

The "unknown" parameters from the inverse problem solution based on a genetic algorithm[19] are given in Table 1. The nine parameters have successfully been determined by the solution of the inverse problem assuming a kinked crack, see Table 1 (left) and two notches, see Table 1 (right) in a finite plate. The SIFs K_I and K_{II} have been calculated subsequently based on the identified parameters.

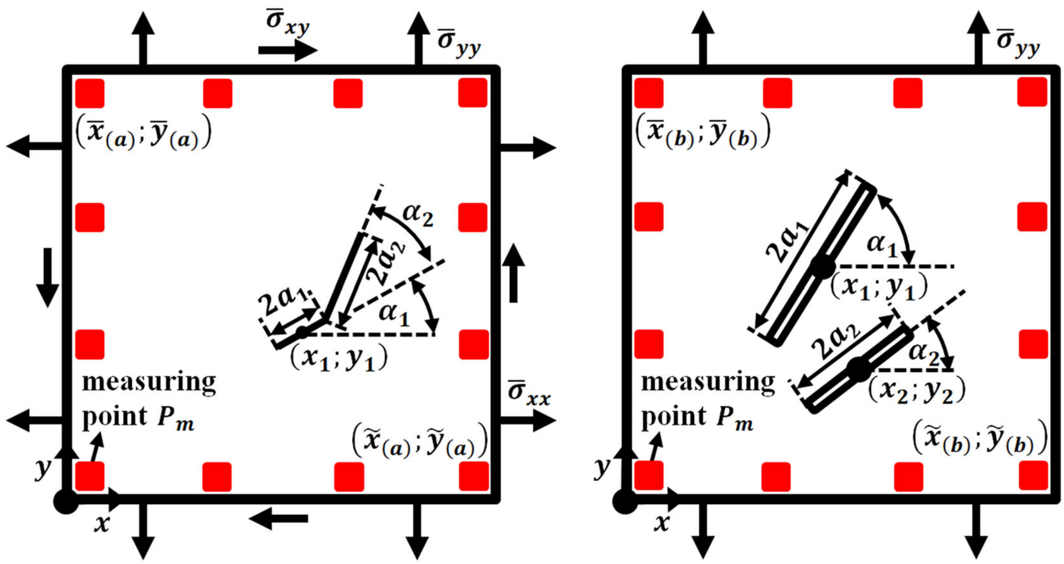

FIGURE 2 Left: Finite plate 30 mm × 30 mm with kinked crack under boundary loads $\overline{\sigma}_{ij}$, $ij = xx, yy, xy$; $P_m(m = 1, ..., 12)$ measuring points and $\left(\overline{x}_{(a)}; \overline{y}_{(a)}\right) = (1;\ 29)$ mm, $\left(\widetilde{x}_{(a)}; \widetilde{y}_{(a)}\right) = (29;\ 1)$ mm; right: finite plate 200 mm × 200 mm with two notches under boundary load $\overline{\sigma}_{yy}$; $P_m(m = 1, ..., 12)$ measuring points and $\left(\overline{x}_{(b)}; \overline{y}_{(b)}\right) = (25;\ 154)$ mm, $\left(\widetilde{x}_{(b)}; \widetilde{y}_{(b)}\right) = (175;\ 46)$ mm

TABLE 1 Left: Results of the crack detection and parameter identification, see Figure 2 (left), $F_{I;II} = K_{I;II}/(\bar{\sigma}_{yy}\sqrt{\pi a_1})$; right: results of the notch detection and parameter identification, see Figure 2 (right)

Parameters	Given	Identified	Parameters	Given	Identified
$\bar{\sigma}_{xx}$ [MPa]	30	30.00	$\bar{\sigma}_{yy}$ [MPa]	20	20
$\bar{\sigma}_{yy}$ [MPa]	90	90.00	a_1[mm]	30	30.45
$\bar{\sigma}_{xy}$ [MPa]	20	19.99	a_2[mm]	20	20.07
a_1[mm]	2	2.02	x_1[mm]	90	90.45
a_2[mm]	3	3.01	y_1[mm]	120	120.09
x_1[mm]	15	15.00	x_2[mm]	120	119.5
y_1[mm]	10	9.99	y_2[mm]	80	79.99
α_1[°]	30	30.34	α_1[°]	40	40.13
α_2[°]	45	44.15	α_2[°]	20	19.63
$F_I(+)$	0.4305	0.4306			
$F_I(-)$	0.8598	0.8593			
$F_{II}(+)$	0.1920	0.2051			
$F_{II}(-)$	0.3931	0.3998			

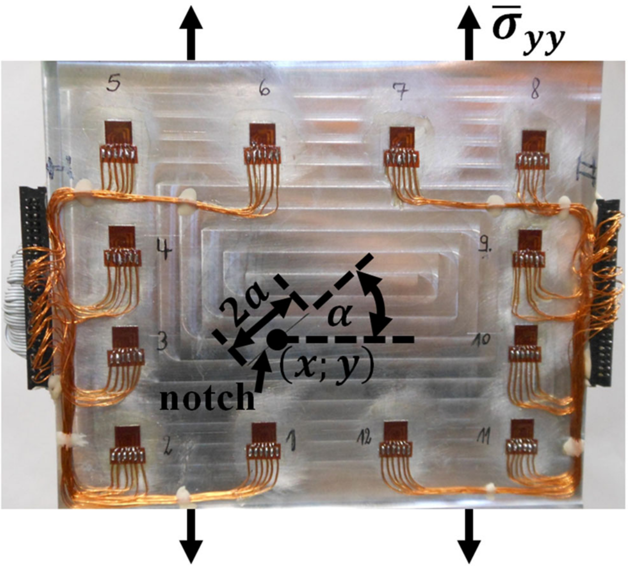

FIGURE 3 Plate 200 mm × 200 mm × 8 mm (*AL-7075*) with a notch under boundary load $\bar{\sigma}_{yy}$ and 12 strain gauges

TABLE 2 Results of the notch detection and parameter identification, see Figure 3

Parameters	Given	Identified
$\bar{\sigma}_{yy}$ [MPa]	18.57	20.44
a [mm]	15	13.72
x [mm]	80	77.24
y [mm]	80	77.28
α [°]	40	36.79

3.2 | Experimental verifications

Besides numerical simulations, real hardware experiments have been performed. Here, loading $\overline{\sigma}_{yy}$, notch length $2a$, notch inclination α, and the position of the notch $(x;\ y)$ are used as testing parameters. We consider a notch in an *Al-7075*-plate as shown in Figure 3. The strain $\varepsilon_{ij}(P_m)$ is measured using strain gauges at points $P_m(m = 1, \ldots, 12)$. The positions of the strain gauges are given in Figure 2 (right). Table 2 shows the obtained experimental results.

4 | CONCLUSIONS

In this work, the concept of distributed dislocations is applied for the detection of cracks or notches and the calculation of SIFs in finite plate structures, where shapes and numbers of cracks have to be known a priori. The method was verified numerically for a finite plate with kinked crack and a finite plate with notches, and experimentally for a plate with a notch. The cracks or notches and loading parameters could be successfully determined by the solution of the inverse problem. In the numerical verification, the input strain data to solve the inverse problem for a finite plate with a kinked crack emerge from the distributed dislocation technique whereas for the finite plate with two notches, the strain data emerge from the FEM. In the real experiment, strain gauges are used to measure the strain at the surface of a plate made of *Al-7075*.

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