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Beyond Ritz-Galerkin: Finite element approximations on a manifold in the configuration space

Christian Schröppel*, Jens Wackerfuß

University of Kassel
Institute of Structural Analysis
Mönchebergstr. 7, 34109 Kassel, Germany
wackerfuss@uni-kassel.de

ABSTRACT

An extension of the Ritz-Galerkin method, based on finding *approximations on a finite-dimensional manifold of functions* (i.e., not a linear subspace) in the infinite-dimensional exact configuration space, will be presented. This new approach is particularly efficient in computing geometrically exact solutions for problems involving large rotations.

The finite element method identifies an approximate solution to a boundary value problem, i.e. a function assigning a position $\boldsymbol{x}^{\mathrm{h}}$ (and therefore, a displacement $\boldsymbol{u}^{\mathrm{h}}$) in the Euclidean space E to every material point X. The exact solution $\boldsymbol{u}(X)$ belongs to an infinite-dimensional function space V. The Ritz-Galerkin method consists of finding a solution $\boldsymbol{u}^{\mathrm{h}}(X)$ in a *linear subspace* V^{h} of V, satisfying the Galerkin orthogonality condition. The interpolant, which determines the solution within a single finite element, is given as a linear combination of shape functions. The Ritz-Galerkin method is particularly efficient if the solution is dominated by translational deformations. The intrinsic nonlinear character of rotations, however, presents serious challenges to finite element formulations based on the Ritz-Galerkin method.

Finite element methods for problems involving large rotations include co-rotational methods (separating rigid-body motions from the remaining deformation components), Cosserat methods (augmenting the solution space by independent rotational values), and curvilinear coordinates (V^{h} is linear with regard to displacements w.r.t. the parameterization of a manifold in E). A major drawback of these approaches is the inability to capture the intrinsic coupling of translational and rotational field values.

Rather than finding an approximate solution on a linear subspace of the exact configuration space, we construct an interpolant that is given as a *finite-dimensional manifold of functions* in the exact configuration space. As in the standard Ritz-Galerkin approach, the interpolant is parameterized by the degrees of freedom associated with the respective finite element. Thus, the interpolant is given as an embedding (a diffeomorphic and thus bijective map) of the space of degrees of freedom into the configuration space, ensuring that compatibility conditions can be formulated and a global finite element system can be set up.

The presentation will include an exposition of the *Logarithmic finite element method* (LogFE method), an implementation of the extended Ritz-Galerkin approach outlined above. The LogFE method uses an interpolant given as the exponential of a linear combination of shape functions (an interpolant of Ritz-Galerkin type) defined on a Lie algebra, resulting in a map (given as a Lie group) that transforms the initial configuration into the current configuration [1]. In particular, the LogFE method allows for the formulation of degrees of freedom based on geometrically exact rotational field values. The results of numerical simulations for 2D and 3D beam models will be presented. Finally, the integration of the isogeometric approach [2] and the LogFE method will be discussed.

References

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[2] Cottrell, J.A., Hughes, T.J.R., Bazilevs, Y., *Isogeometric analysis. Towards integration of CAD and FEA*, John Wiley & Sons, Chichester 2009.