

Influence of Mean Stress on Lifetime Prediction of Adhesive Bonds

Christian Köster¹ and Anton Matzenmiller^{1,*}

¹ Institute of Mechanics, Dept. of Mechanical Engineering, University of Kassel, Mönchebergstr. 7, 34125 Kassel, Germany

The importance of adhesive bonds in the field of joining technology is increasing due to their advantageous properties. One of these attributes is the possibility to reduce the weight compared to welded joints in group of components. For a safe design, the knowledge of the lifetime of these components is essential. Various methods are used for the prediction of the lifetime under cyclic loading. On the one hand there are test-based methods, on the other hand simulation-based approaches. Test-based ones are often time-consuming and cost-intensive. In addition, a change in boundary conditions requires new tests. Using a simulation-based method, it is possible to analyse new boundary conditions rapidly. The following contribution shows a damage model approach that is able to predict the lifetime of an adhesive bond with high accuracy, by taking into account the main influencing factors. First, various lifetime influencing factors are presented. Afterwards, the model approach is extended in regard to the consideration of the important mean stress influence. Finally, the model approach is validated by the comparison of the model-based lifetime prediction to the one from test data under multiaxial loading.

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1 Lifetime influencing factors

A valid lifetime prediction model for cyclic loading has to consider the main mechanical factors influencing the lifetime. Some of these can be described by the time-dependent stress function σ :

$$\sigma(t) = \sigma_m + \sigma_a \sin(2\pi ft + \phi) \quad (1)$$

Included are the mean stress σ_m , the stress amplitude σ_a , the frequency f and the phase angle ϕ . Furthermore, several directions of stresses and their combinations are conceivable. Additional influencing factors are the difference between tensile and compressive stresses and the order of stress amplitudes. An existing damage model developed in [1] and [2] for the calculation of the lifetime of adhesive bonds is able to capture the effects mentioned, but the important influence of mean stress is not explicitly considered.

A central definition for the description of a stress cycle and the evaluation of the mean stress influence is the stress ratio R . This quantity is defined by the ratio of the minimum σ_{\min} and the maximum σ_{\max} stress during a cycle:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad (2)$$

The range of values of R in the case of cyclic loading is $-1 \leq R \leq 1$. The case $R = -1$ results in an alternating loading, i.e. $\sigma_{\min} = -\sigma_{\max}$. The consequence of $R = 1$ is $\sigma_{\min} = \sigma_{\max}$ and, thus, a constant loading. The quantity R also allows the calculation of the mean stress σ_m when the stress amplitude σ_a is known, due to the following expression:

$$\sigma_m = \sigma_a \frac{1 + R}{1 - R} \quad (3)$$

Concerning the mean stress, $R = -1$ results in a mean stress of zero, while constant stress amplitudes σ_a and increasing values of R lead to growing mean stresses σ_m .

The equations (2) and (3) can be applied for normal and shear stresses. Further important definitions are the expressions for the calculation of the amplitude and the mean stress, depending on the maximum σ_{\max} and minimum σ_{\min} values of the stress history during a cycle. The relationships are as follows:

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} \quad (a), \quad \sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \quad (b) \quad (4)$$

By inserting the equations (2) and (4a) into equation (3), it can be shown that this expression is also defined for $R = 1$ and is equal to the expression in (4b).

As stated, the influence of the mean stress is not explicitly considered in the model of [1] and [2]. However, using test data from [3] and [4], it can be shown that this factor has a significant influence on the lifetime of adhesive bonds and must, therefore, be taken into account in the model. Figure 1 illustrates the influence of an increasing stress ratio R using the SN-curves of the mentioned test data. The left part of figure 1 demonstrates the impact of an increasing value of R in the case of pure shear stress. The right part shows the case of pure tensile stress. In both sections of figure 1, it can be seen that an increase of the stress ratio R leads to a decrease of the fatigue strength. This is evident by the fact that as R increases, a smaller amplitude is required to reach the same lifetime. This behaviour is due to the higher mean stress as a result of a higher stress ratio. Additionally, this figure motivates the need for a consideration of the mean shear and normal stress.

* Corresponding author: e-mail post-structure@uni-kassel.de, phone +49 (0)561 804 2043, fax +49 (0)561 804 2720



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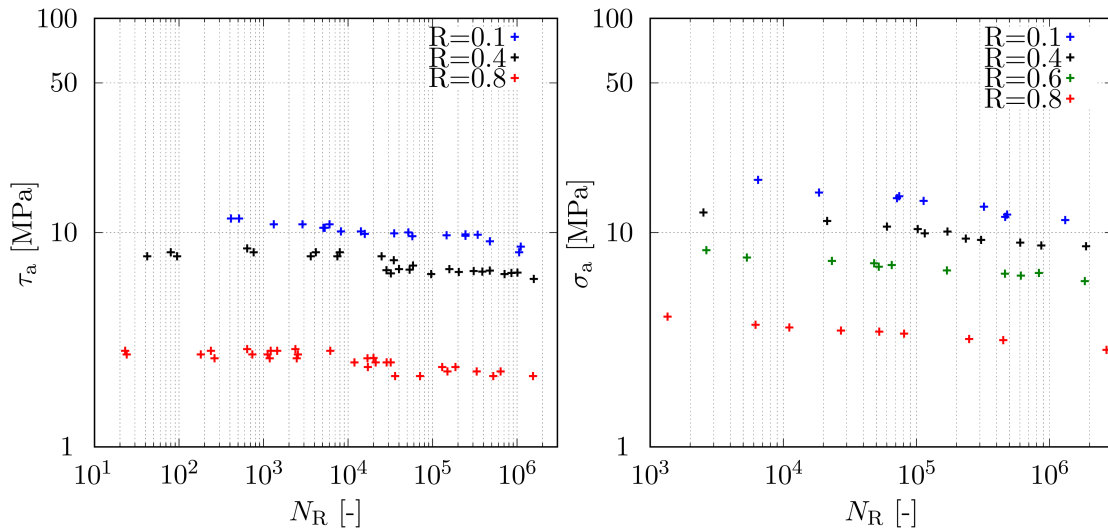


Fig. 1: Test data-based analysis of influence of stress ratio on fatigue strength left part: pure shear stress with test data from [3]; right part: pure tensile stress with test data from [4]

2 Damage modeling for adhesive bonds

The lifetime prediction model, developed in [2], is based on the calculation of damage of a thin adhesive layer due to cyclic loading. This lifetime is reached when the damage variable D takes on a value of $D = 1$. The model mentioned above is based on the superposition of a creep damage rate \dot{D}_c and a cycle-dependent fatigue damage part dD_f/dN according to [5] as follows:

$$dD = \dot{D}_c dt + \frac{dD_f}{dN} dN \quad (5)$$

For \dot{D}_c and dD_f/dN suitable approaches are chosen to describe the damage behaviour of an adhesive bond. In [1] and [2] the creep damage evolution \dot{D}_c is modeled according to [6] by the exponential approach:

$$\dot{D}_c = \frac{1}{c_0} \left(\frac{\langle \sigma_{eqc} - \sigma_{dc} \rangle}{\sigma_{ref}(1-D)} \right)^n \quad (6)$$

This expression contains the creep limit σ_{dc} and the two further model parameters σ_{ref} and n . A consistent unit is created by choosing the parameter $c_0 = 1$ s, [2]. The equivalent creep stress σ_{eqc} is used to obtain a stress value in the case of multiaxial stress states and serves as the damage controlling variable. The following invariant-based equivalent stress approach according to the work of SCHLIMMER [7] was modified and successfully used in [2]:

$$\sigma_{eqc} = \sqrt{\langle b_{1c} \langle t_n \rangle^2 + b_{2c} t_n + \bar{t}_t^2 + \bar{t}_b^2 \rangle} \quad \text{with} \quad \bar{t}_t = t_t(1 + b_3 \langle -R_t \rangle) \quad , \quad \bar{t}_b = t_b(1 + b_3 \langle -R_b \rangle) \quad (7)$$

The stress components t_n , t_t and t_b result from the assumption of a transverse strain constrained stress state for thin adhesive layers [1]. This stress state has three independent components, the normal stress component t_n and two shear stress components t_t and t_b . The modifications to replace t_t with \bar{t}_t and t_b with \bar{t}_b were made to obtain valid lifetime prediction under alternating loading [2]. For this purpose, $b_3 = 1$ was assumed in [2]. The equivalent stress approach also includes two model parameters b_{1c} and b_{2c} to model the impact of the normal stress component. The MACAULAY-bracket around the quadratic normal stress component t_n is used to account the difference between tensile and compressive stresses. Due to this modification, negative values for the radicand are possible, which result in numerical problems. This is prevented by using a second MACAULAY-bracket around the entire radicand. This case only occurs if the normal stress component t_n is smaller than 0 and leads to an equivalent creep stress of $\sigma_{eqc} = 0$. In combination with equation (6), the damage growth per time step is $\dot{D}_c = 0$ and results in an infinite lifetime. This assumption is based on the observation in [8] that the superposition of a shear alternating stress with a compressive stress leads to an increase of the lifetime.

The parameters n , σ_{ref} , σ_{dc} , b_{1c} and b_{2c} , occurring in the equations (6) and (7), were determined based on test data from [8]. The resulting set of creep parameters is shown in the following table:

Table 1: Parameter of creep damage and creep equivalent stress approach from [2]

σ_{dc} [MPa]	σ_{ref} [MPa]	n [-]	b_{1c} [-]	b_{2c} [MPa]
0	51	19	0.5	12

The fatigue damage approach is based on the work of LEMAITRE and CHABOCHE. The expression dD_f/dN in equation (5) is modeled by the term:

$$\frac{d\tilde{D}}{dN} = \frac{\tilde{D}^\alpha}{(1 - \alpha)N_{Rf}} \quad \text{with} \quad \tilde{D} = 1 - (1 - D)^{\beta+1} \tag{8}$$

according to [9]. In equation (8), the model quantities N_{Rf} and α and the parameter β occur. N_{Rf} represents an analytical approach to describe a SN-curve and can be defined, for example, by the BASQUIN-equation [9]. α is used for the consideration of the amplitude order, resulting in a model that is able to describe non-linear damage accumulation. The change of variable from D to \tilde{D} is proposed in [10] to better fit a simulated damage curve to test data. For the model quantity α , the following equation was proposed in [11]:

$$\alpha = 1 - a \left\langle \frac{A_{II} - A_{II}^*}{\sigma_{ult} - \sigma_{eqmax}} \right\rangle \quad \text{with} \quad A_{II}^* = \sigma_{df}(1 - 3b_{1m}\sigma_H) \tag{9}$$

Here, A_{II} represents the stress parameter, A_{II}^* describes the influence of mean stress on the fatigue limit σ_{df} due to model parameter b_{1m} and the mean hydrostatic pressure $\sigma_H = (I_{1max} + I_{1min})/6$. σ_{ult} is a material specific model parameter, which has to be identified by test data. The quantity σ_{eqmax} is the maximum value of an equivalent stress approach during a cycle. The parameter a is introduced in [10] as a fitting parameter to describe the damage evolution and can only be obtained from measured damage curves. Due to the quantities A_{II} and σ_{eqmax} and their changing values at different load cycles, the model is able to calculate non-linear damage accumulation. The original model equation for A_{II} according to [11] is based on the deviatoric stress tensor and is shown in the following.

$$A_{II} = \frac{1}{2} \sqrt{\frac{3}{2} (\sigma_{maxij}^{dev} - \sigma_{minij}^{dev}) (\sigma_{maxij}^{dev} - \sigma_{minij}^{dev})} = \frac{1}{2} \sqrt{\frac{3}{2} \Delta\sigma_{ij}^{dev} \Delta\sigma_{ij}^{dev}} = \frac{1}{2} \Delta\sigma_{vM} = \sigma_{vMa} \tag{10}$$

The expression for A_{II} includes the maximum σ_{maxij}^{dev} and minimum σ_{minij}^{dev} values of each component of the deviatoric stress tensor. The simplification of this expression shows that this expression is equal to the half of the difference of the maximum and minimum values of the VON-MISES equivalent stress. According to equation (4a), this is equal to the amplitude of the VON-MISES equivalent stress σ_{vMa} .

The consideration of the mean stress influence is based on a suitable approach for N_{Rf} from equation (8). The approach used in this contribution can be calculated from the following fatigue damage evolution proposed in [11]:

$$dD_f = [1 - (1 - D_f)^{\beta+1}]^\alpha \left[\frac{A_{II}}{M(\sigma_H)(1 - D_f)} \right]^\beta dN \tag{11}$$

By separating and integrating with respect to the integration limits $N = 0$ to $N = N_R$, as well as $D_f = 0$ to $D_f = 1$, the following equation can be obtained:

$$N_{Rf} = \frac{1}{(1 - \alpha)(1 + \beta)} \left(\frac{A_{II}}{M(\sigma_H)} \right)^{-\beta} \tag{12}$$

This expression includes a mean stress dependent function M which is defined in [11] by:

$$M(\sigma_H) = M_0(1 - 3b_{2m}\sigma_H) \tag{13}$$

as a function of two model parameters M_0 and b_{2m} and the mean hydrostatic pressure σ_H . The use of σ_H as the mean stress component has the consequence that shear mean stresses can not be considered. For this reason, a modification is necessary, but the basic structure of equation (13) is maintained. The modified expression for the consideration of mean stress σ_{mean} is presented as follows:

$$M(\sigma_{mean}) = M_0(1 - 3b_{2m}\sigma_{mean}) \tag{14}$$

The mean stress σ_{mean} included is described by the following expression:

$$\sigma_{mean} = \frac{\sigma_{eqfmax} + \sigma_{eqfmin}}{2} (1 - \langle -R_\sigma \rangle)(1 - \langle -R_t \rangle) = \sigma_{eqfmean} (1 - \langle -R_\sigma \rangle)(1 - \langle -R_t \rangle) \tag{15}$$

The dependency of this expression on the normal stress ratio R_σ and shear stress ratio R_t ensures that valid mean stress values are calculated in the case of alternating proportional loadings ($R_\sigma = -1$ or $R_t = -1$). The values occurring therein are the minimum and maximum values of the equivalent fatigue stress σ_{eqfmin} and σ_{eqfmax} during a load cycle. The calculation of

the equivalent fatigue stress σ_{eqf} is based on the expression of the creep equivalent stress from equation (7) and is defined as follows:

$$\sigma_{\text{eqf}} = \sqrt{\langle b_{1f} \langle t_n \rangle^2 + b_{2f} t_n + \bar{t}_t^2 + \bar{t}_b^2 \rangle} \quad (16)$$

In accordance to the creep equivalent stress approach, there are two additional model parameters b_{1f} and b_{2f} to scale the normal stress component. Also included are the mentioned MACAULEY-brackets and the modified stress components \bar{t}_t and \bar{t}_b . The stress parameter A_{II} from equation (10) uses the VON-MISES equivalent stress to transfer a multiaxial stress state into a scalar quantity. However, this type of equivalent stress is not suitable for predicting the lifetime of an adhesive bond and has to be modified. For this reason, the equivalent stress approach from equation (16) and their maximum and minimum values during a cycle are used here as well. However, the structure of the expression from equation (10) as an amplitude of an equivalent stress approach is retained. This results in the following relationship:

$$A_{\text{II,mod}} = \frac{1 + \langle -R_\sigma \rangle}{(1 + K_m R_\sigma)} \frac{1}{2} (\sigma_{\text{eqfmax}} - \sigma_{\text{eqfmin}}) = \frac{1 + \langle -R_\sigma \rangle}{(1 + K_m R_\sigma)} \sigma_{\text{eqfa}} \quad (17)$$

Equation (17) includes the maximum and minimum values of equivalent fatigue stress σ_{eqf} . The term $(\sigma_{\text{eqfmax}} - \sigma_{\text{eqfmin}})/2$ can also be interpreted as the amplitude of the equivalent stress σ_{eqfa} . The factor in front of the equivalent stress amplitude is required to fit the model response to the test data. In this factor, an additional model parameter K_m occurs, which has to be determined by test data. In addition to the adjustment of the stress parameter A_{II} and the mean stress σ_{mean} , the parameter α and A_{II}^* from equation (9) are also modified and defined as follows:

$$\alpha_{\text{mod}} = 1 - a \left\langle \frac{A_{\text{II,mod}} - A_{\text{II,mod}}^*}{\tau_u - \sigma_{\text{eqfmax}}} \right\rangle \quad \text{with} \quad A_{\text{II,mod}}^* = \sigma_{\text{df}} (1 - 3b_{1m} \sigma_{\text{mean}}) \quad (18)$$

The modification of α involves the use of the stress parameter $A_{\text{II,mod}}$ from equation (17), the replacement of σ_{ult} by $\tau_u = 37 \text{ MPa}$, [2], and the substitution of σ_{eqmax} by the maximum of equivalent stress σ_{eqfmax} which is calculated by the approach from equation (16).

In addition, the adjusted equations (14), (15), (17) and (18) are used to determine the quantity N_{Rf} . This results in the following expression:

$$N_{\text{Rf}} = \frac{1}{(1 - \alpha_{\text{mod}})(1 + \beta)} \left(\frac{A_{\text{II,mod}}}{M(\sigma_{\text{mean}})} \right)^{-\beta} \quad (19)$$

The implementation of the described model equations is based on the solution of the differential equation from equation (5). In [9], a sequential procedure for the solution of this equation is proposed. In the first step the creep damage from equation (6) is calculated numerically, in [2] and [12] the BDF-2 algorithm is used. The resulting problem is solved by using the NEWTON procedure in each time step. The second part is the calculation of a fatigue damage increment ΔD_f based on a recursion equation. The use of such a recursion equation results in the fatigue damage being calculated only once per cycle [9]. In [2] and [12] the fatigue damage is calculated if the maximum of the stress in the cycle is reached. This procedure is also used in this contribution.

The recursion equation results from the separation and integration of the fatigue damage approach from equation (8). The integral to be solved is described in [12] as follows

$$\frac{d\tilde{D}}{dN} = \frac{\tilde{D}^\alpha}{(1 - \alpha)N_{\text{Rf}}} \Rightarrow \int_{\tilde{D}_{f1}}^{\tilde{D}_{f2}} \frac{1}{\tilde{D}^\alpha} d\tilde{D} = \int_N^{N+1} \frac{1}{(1 - \alpha)N_{\text{Rf}}} dN \quad (20)$$

The integration limits are the fatigue damage values \tilde{D}_{f1} before and \tilde{D}_{f2} after the occurrence of the stress maximum. Due to the calculation of the fatigue damage only once per cycle this fatigue damage values are representative for the entire cycle. Solving the equation (20) and inserting the integral limits results in the following expression:

$$\tilde{D}_{f2} = \left[\tilde{D}_{f1}^{1-\alpha} + \frac{1}{N_{\text{Rf}}} \right]^{\frac{1}{1-\alpha}} \quad (21)$$

Taking into account the change of variables from \tilde{D} to D from equation (8) and replacing α by α_{mod} from equation (18), the following equation for calculating the fatigue damage increment ΔD_f is obtained:

$$\Delta D_f = 1 - \left(1 - \left(\left(1 - (1 - D)^{\beta+1} \right)^{1-\alpha_{\text{mod}}} + \frac{1}{N_{\text{Rf}}} \right)^{\frac{1}{1-\alpha_{\text{mod}}}} \right)^{\frac{1}{\beta+1}} - D \quad (22)$$

Where D is the current damage value, consisting of the damage value before the calculation of the new fatigue damage increment and the current creep damage. In addition, the model quantities N_{Rf} and α_{mod} from equations (18) and (19) as well as the model parameter β are included.

3 Parameter identification and validation of the damage model

The implementation of the equations from chapter 2 and their numerical solution enable the identification of the model parameters. The determination is carried out under consideration of the creep parameters from table 1. The first identification step is to calculate the fatigue damage parameters M_0 , b_{2m} and β from equations (14), (15) and (19) using uniaxial test data from [8] and [13] under pure shear stress with three different stress ratios $R_t = -1$, $R_t = 0.1$ and $R_t = 0.4$. Due to the assumption of $\sigma_{df} = 0$ in [2], the exact value of b_{1m} from equation (18) is not of interest. As mentioned in chapter 2, the determination of the parameter a is only possible on the basis of measured damage curves, which are not available for the adhesive under consideration. However, further investigations have shown that the parameter a from equation (18) can be set arbitrarily. The other model parameters are adjusted in such a way that the prediction quality is not significantly influenced. For this reason, $a = 1$ is assumed according to [12].

The target values of the optimization process are the double-logarithmic number of cycles from the SN-curves of the mentioned test data sets. An analytical solution of this identification problem is not possible, therefore the software LS-OPT [14], is used. LS-OPT optimizes the model parameters by minimizing the mean squared error between the simulation and the test data. Based on the determined fatigue damage parameters, the equivalent stress parameters b_{1f} and b_{2f} from equation (16) and K_m from equation (17) are next calculated. The procedure is the same as for the fatigue parameters, besides that test data sets under pure normal load from [13] are used. The stress ratios considered are also $R_\sigma = -1$, $R_\sigma = 0.1$ and $R_\sigma = 0.4$. The resulting set of parameters are listed in the following table:

Table 2: Parameter of fatigue damage approach

M_0 [MPa]	b_{2m} [MPa ⁻¹]	β [-]	b_{1f} [-]	b_{2f} [MPa]	K_m [-]	σ_{df} [MPa], [2]	a [-], [12]
55.929	$2.84 \cdot 10^{-3}$	10.35	0.477	16.89	-0.636	0	1

The values in table 2 are used in combination with the creep damage parameters from table 1 to validate the model approach. Since the identification is carried out on uniaxial test data sets, the validation is performed by comparing the simulation results with a test data set of a multiaxial loading, which is characterized by the stress ratios of the individual loading and the ratio between the shear stress and the normal stress amplitude $\tau_a/\sigma_a = t_{ta}/t_{na}$. Figure 2 shows the comparison of the simulation (red) and the test data (black) from [8] in the case of a multiaxial loading with the stress ratios $R_t = R_\sigma = 0.1$ and an amplitude ratio of $\tau_a/\sigma_a = 2$. The comparison of test data and simulation shows a slight difference in the slope of the SN-

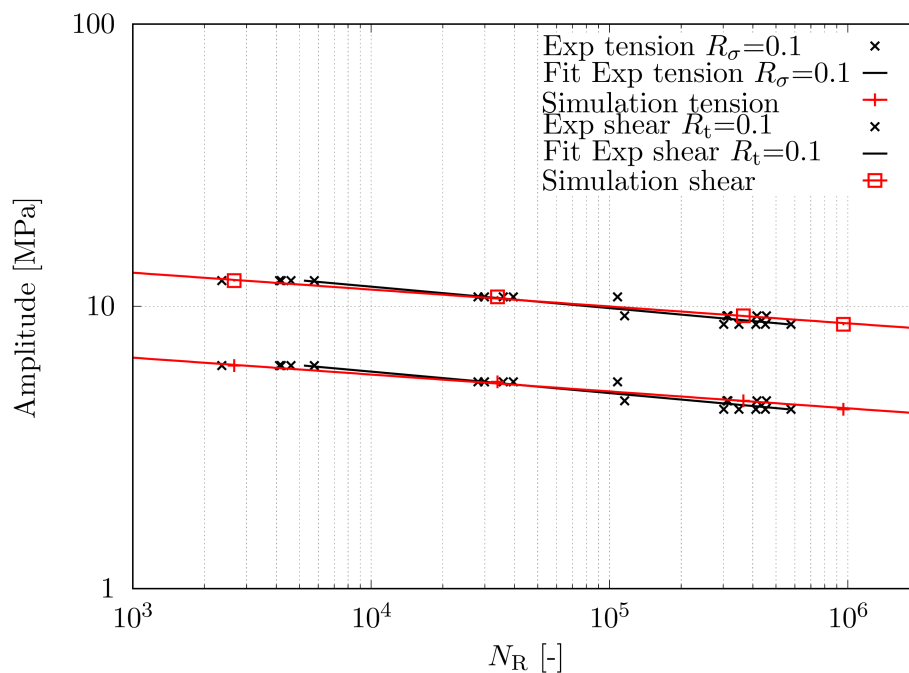


Fig. 2: Comparison of the simulated lifetime with test data from [8] under a multiaxial loading with the stress ratios $R_\sigma = R_t = 0.1$ and amplitude ratio $\tau_a/\sigma_a = 2$

curves. This difference results in an overestimation of the lifetime in the smallest load case. A possible reason for the deviation is the use of test data from [8] for the validation, although test data from [13] were primarily used for the identification of the parameters. All in all, this comparison shows a good agreement between test data and simulation.

4 Summary and outlook

This contribution presents a damage model that can be used for the prediction of the lifetime of an adhesive bond with high accuracy. The modeling of damage is based on [2] and the work of LEMAITRE and CHABOCHE, [9] and [11]. The model from [11] has the advantage of considering the mean stress influence by employing additional model parameters. Due to its development for steel applications, it is necessary to modify the original model for the calculation of the lifetime of an adhesive bond. Thereby, the stress parameter A_{II} and the calculation of the mean stress σ_{mean} are adjusted. In addition, a parameter identification method was developed and presented. A validation of the model approach is shown based on the comparison of simulation and multiaxial test data, which indicates a good agreement. The next work is related to the further validation of the approach by implementing it in a commercial Finite-Element analysis software (FEA) and evaluating the prediction quality for more complex component geometries.

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