

Notizen

Values for the Electron Screening in Muonic Atoms for all Z

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The electron screening correction in the X-ray transitions in muonic atoms is calculated within a relativistic SCF Hartree-Fock procedure for many transitions and all Z .

In view of the increased accuracy of the measurements of the transition energies in muonic atoms exact values for all corrections have to be calculated and tabulated.

The main contribution in heavy atoms is the effect of the extended nucleus¹. The other corrections

are the vacuum polarisation including the higher order terms^{2,3}, the Lamb-shift⁴, the nuclear polarisation^{5,6} and the relativistic reduced mass correction⁷. First estimations of the electron shielding contribution have been given by Cohen⁸ and Fricke⁹.

There is a great uncertainty in the electron shielding correction which is the result of the fact that we do not know how many electrons remain bound and in which states they are when the muonic cascade proceeds. From purely energetic arguments one finds for example for lead that at least 30 electrons remain in the atom when the muon is in the state $n = 15$ and about 15 when the muon has reached $n = 8$ even under the assumption that the total energy of the bound muon is released by ejection of electrons only. On the other hand the two inner K-electrons which contribute more than 80% to the electron density inside the radius of the muon orbits with $n < 14$ and hence are the bulk of the shielding are

Screening of Muons by Electrons for various transitions

(for 1, 2 and 10 electrons)

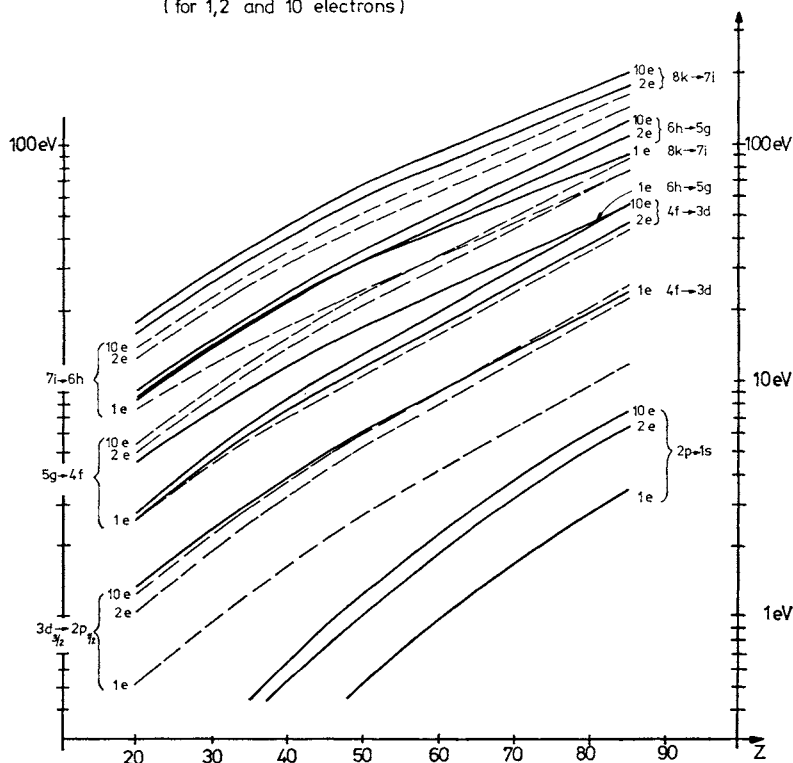


Fig. 1. The electron screening contribution for various transitions for the elements $Z=20$ to $Z=85$ in the presence of 1, 2 and 10 electrons.

bound with nearly 100 keV and can hence be ejected only in muonic Auger transitions with $n = 7 \rightarrow 6$ or deeper, transitions for which radiative processes already dominate over the Auger effect.

The proper method to calculate the electron shielding is to do fully selfconsistent relativistic atomic calculations taking the muon and m electrons into account. This method is good because the motion of the inner electrons at least up to $n = 4$ is fast in comparison to the lifetime of the muonic states. Therefore the most adequate physical quantities in such a quantum mechanical description are the difference of the total energies for the whole system muon plus electrons. The total energy is given by the expression

$$E_T = \sum_i \langle i | \text{KE} + V_N(r) | i \rangle + \sum_{i < j} \left(\left\langle ij \left| \frac{1}{r_{ij}} \right| ij \right\rangle - \left\langle ji \left| \frac{1}{r_{ij}} \right| ij \right\rangle \right) + \langle \mu | \text{KE} + V_N(r) | \mu \rangle + \sum_i \left\langle i \mu \left| \frac{1}{r_{i\mu}} \right| i \mu \right\rangle$$

calculated with the selfconsistent wave functions $|i\rangle$ for the electrons and $|\mu\rangle$ for the muon. KE is the operator of the kinetic energy and V_N is the extended nuclear potential. Because electron and muon are non-identical particles there is no exchange term in the electron muon interaction.

All other methods of computation by taking only the differences of the muonic energy eigenvalues or even by taking non self-consistent electron densities to compute the shielding effect are only approximations. Nevertheless for the lower transitions these approximations yield good values for this correction, but the higher the interesting muonic levels are the higher is the correction of the electron screening and the greater is the influence due to the selfconsistent calculation¹⁰.

In Fig. 1 we present the results of fully selfconsistent Dirac-Slater calculations for the electron shielding contribution taking into account 1, 2 and 10 electrons for nearly all Z . The increase of the shielding contribution for more than 10 electrons is only of the order of 3 to 5%. The most reasonable value for the actual shielding contribution will be the value for 10 electrons.

The higher the muonic transitions are, the higher is the contribution from the electron screening and the smaller is the fine structure splitting.

Tab. I. The transition energies for some finestructure components in muonic lead from the main quantum number $n=8$ to $n=7$ without the presence of electrons and with 10 electrons. The difference is the electron shielding contribution.

| Transition | without electrons keV | with 10 electrons keV | electron screening eV |
|---|-----------------------|-----------------------|-----------------------|
| 8k _{15/2} → 7i _{13/2} | 90.724 | 90.536 | 188 |
| 8i _{13/2} → 7h _{11/2} | 90.939 | 90.735 | 204 |
| 8h _{11/2} → 7g _{9/2} | 91.326 | 91.108 | 218 |
| 8g _{9/2} → 7f _{7/2} | 91.892 | 91.659 | 233 |
| 8f _{7/2} → 7d _{5/2} | 92.628 | 92.381 | 247 |
| 8d _{5/2} → 7p _{3/2} | 87.338 | 87.092 | 246 |
| 8p _{3/2} → 7s _{1/2} | 61.699 | 61.502 | 197 |

To give an example we present in Table 1 the transition energies for various finestructure components between the main quantum numbers $n=8$ and $n=7$ for lead as well as the resulting electron screening corrections. This drastically shows that the electron screening varies as much as 30% for different finestructure components and thus has to be calculated separately for every component. These differences are especially important when the finestructure cannot be resolved anymore experimentally.

¹ See for example K. W. Ford and G. A. Rinkler, Phys. Rev. C **7**, 1206 [1973].

² B. Fricke, Z. Physik **218**, 495 [1969].

³ J. Blomquist, Nucl. Phys. B **4**, 95 [1972].

⁴ R. C. Barrett, Phys. Lett. **29 B**, 93.

⁵ M. Y. Chen, Phys. Rev. C **1**, 1167 and C **1**, 1176 [1970].

⁶ K. Tanabe, Phys. Rev. A **3**, 1282 [1971].

⁷ B. Fricke, Phys. Rev. Lett. **30**, 119 [1973].

⁸ S. Cohen, UCRL-8389 [1958].

⁹ B. Fricke, Lett. Nuovo Cim. **2**, 859 [1969].

¹⁰ B. Fricke, J. T. Waber, and V. L. Telegdi, reprint Northwestern University 1972.