On the Screening of Muons by Electrons in Muonic Atoms (*).

B. FRICKE

Institut für Theoretische Physik der Universität Frankfurt am Main - Frankfurt am Main

(ricevuto il 6 Novembre 1969)

In recent years the accuracy of the measurements of the muonic X-rays increased appreciably so that all effects which change the transition energies by a keV and even less become important [1,2]. Exact calculations have been given for the effect of the finite nuclear size, the nonspherical components of the charge distribution [3], the nuclear polarization [4], the vacuum polarization and their higher-order contributions [5] as well as the Lamb-shift [6].

First calculations of the electron screening were given by COHEN [7]. He found a potential due to screening of the muon by the electrons of the form $a - br^{2\sigma}$, where $\sigma = (1 - Z^2\alpha^2)^{1/2}$ $(a, b > 0)$. The term a shifts all levels by a constant positive energy (several keV) and the second term lowers the muonic $1s$ level of Pb by 5 eV and the $5g$ level by 190 eV, so that all transition energies will be reduced by a small amount, typically of the order of 80 eV for the $5g$-$4f$ transition. Some other calculations were done by taking into account the Thomas-Fermi electronic charge distribution [8], hydrogen [2] or self-consistent electronic wave functions [9].

We have calculated the electron screening in a total self-consistent way by using a Hartree-Fock-Slater program [10] *in which the electrons as well as the muon are included.*

(*) This work has been supported by the Deutsche Forschungsgemeinschaft and the Bundesministerium für Wissenschaftliche Forschung.

(1) H. L. ANDERSON, S. K. HARGROVE, E. P. HINCKS, J. D. ANDREW, R. J. MCKEE, R. D. BARTON and D. KESSLER: to be published.

(2) G. BACKENSTOSS, L. TAUSCHER, S. CHARALAMBUS, H. DANIEL, H. KOCH, CH. V. D. MALSBURG, G. BILZ and H. SCHMIDT: *Measurement of the vacuum polarization in muonic atoms*, contributed paper to the *III International Conference on High-Energy Physics and Nuclear Structure, 8-12 Sept. 1969, New York*.

(3) W. PIEPER and W. GREINER: *Nucl. Phys.*, A **109**, 539 (1968).

(4) R. K. COLE: *Phys. Rev.*, **117**, 164 (1969).

(5) B. FRICKE: *Zeits. Phys.*, **218**, 495 (1969).

(6) R. C. BARRETT: *Phys. Lett.*, **28** B, 93 (1968).

(7) S. COHEN: University of California Radiation Laboratory Report UCRL-8389 (1958).

(8) H. BACKE: Thesis Darmstadt (1969).

(9) H. L. ANDERSON: contributed paper, *III International Conference on High-Energy Physics and Nuclear Structure, New York, Sept. 1969*.

(10) J. T. WABER and D. T. CROMER: to be published. We are grateful for Dr. J. T. WABER for communicating us the computer program.

The energy eigenvalues are calculated by numerical integration of the Dirac equation with a potential due to the proton charge distribution (Fermi type) of the nucleus, the static Coulomb interaction of the electrons and the muon as well as the exchange potential of the electrons ([11]). The self-consistency criterion, the relative change of the total potential, are of the order $\Delta V/V < 10^{-6}$ and the energy eigenvalues of the muon are calculated with an accuracy $\Delta E/E < 10^{-7}$ and for the electrons $< 10^{-6}$.

The fundamental difficulty in the calculation of the electron screening is the following. Because of the Auger effect (nonradiative transitions of the muon) the atom becomes very highly ionized as long as the muon is in highly excited levels. Thus one does not know in general how many atomic electrons are left and which atomic levels are occupied by them. The unknown recapturing process during the muon cascade worsens the situation even more. Nevertheless some cascade calculations ([12]) suggest that most electron shells (not only the inner ones) are refilled very soon so that the degrees of ionization might be quite small, especially if the muon has already cascaded down to the lower levels. The E^{+3} energy-dependence of the transition probability may indicate this behaviour as well.

Because of this uncertainty we have therefore calculated the binding energies of the muon in the 1s up to the 8k level for 80, 46, 28 (the filled inner 3 shells), 10 (the inner 2 shells), 2 (the filled K shell) and zero electrons in the case of ^{208}Pb. The nuclear parameters of Anderson *et al.* ([1]) have been used. The differences in the binding energy

TABLE I. – *The muon binding energies for lead (in* keV*) for different levels and different numbers of outer electrons.* For zero electrons the total binding energy is given, for all the other cases we show only the shift due to the electron screening.

Level	Number of electrons					
	0	2	10	28	46	81
$1s_{1/2}$	– 10 525.456	5.383	10.275	14.132	15.804	17.190
$2p_{1/2}$	– 4 782.313	5.381	10.272	14.128	15.800	17.185
$3d_{3/2}$	– 2 162.653	5.361	10.249	14.105	15.776	17.162
$4d_{3/2}$	– 1 213.989	5.300	10.177	14.031	15.702	17.086
$4f_{5/2}$	– 1 197.504	5.320	10.201	14.055	15.726	17.112
$4f_{7/2}$	– 1 188.438	5.319	10.200	14.053	15.725	17.107
$5f_{5/2}$	– 766.442	5.220	10.083	13.933	15.604	16.985
$5g_{7/2}$	– 761.806	5.255	10.124	13.976	15.646	17.032
$6g_{7/2}$	– 529.156	5.110	9.954	13.801	15.471	16.848
$6h_{9/2}$	– 527.603	5.160	10.013	13.861	15.531	16.918
$7i_{11/2}$	– 387.085	5.036	9.866	13.710	15.379	16.765
$8k_{13/2}$	– 296.137	4.881	9.684	13.523	15.190	16.576

for the muon in the presence of a number of electrons and without any electron (naked nucleus) are presented in Table I. In the case of the naked nucleus the binding energies are given absolutely. The absolute shifts for the different electron configurations are quite large (in the case of 81 electrons 17 keV). Nevertheless the absolute value is nearly

([11]) J. C. SLATER: *Phys. Rev.*, **81**, 385 (1951).
([12]) D. KESSLER: private communication.

equal for the muon in the different levels. This is at first astonishing because a muon in a very high level will be screened by much more electrons than for example the muon in the 1s level. Both effects, the increase of the screening electrons and the decrease of the eigenvalues with increasing principal quantum number n nearly cancel each other.

Therefore the observable effect in the transition energies is very small. This can be seen from Table II where the changes due to electron screening for various transitions

TABLE II. – *The observable effect of the electron screening (in* eV*) for lead for various transitions and different numbers of electrons.*

Transition	Number of electrons					
	0	2	10	28	46	81
$2p_{1/2} \to 1s_{1/2}$	5734.143	$-$ 2	$-$ 3	$-$ 4	$-$ 4	$-$ 5
$3d_{3/2} \to 2p_{1/2}$	2619.660	$-$ 20	$-$ 23	$-$ 23	$-$ 24	$-$ 23
$4f_{5/2} \to 3d_{3/2}$	965.144	$-$ 41	$-$ 48	$-$ 50	$-$ 50	$-$ 50
$5g_{7/2} \to 4f_{5/2}$	435.698	$-$ 65	$-$ 77	$-$ 79	$-$ 80	$-$ 80
$6h_{9/2} \to 5g_{7/2}$	234.203	$-$ 95	$-$ 111	$-$ 115	$-$ 115	$-$ 114
$7i_{11/2} \to 6h_{9/2}$	140.518	$-$ 124	$-$ 147	$-$ 151	$-$ 152	$-$ 153
$8k \to 7i$	90.948	$-$ 155	$-$ 182	$-$ 187	$-$ 189	$-$ 189

and various electron configurations are listed. These values indicate that the calculation using perturbation techniques with self-consistent wave functions ([9]) from other calculations ([13]) are nearly identical with our exact calculation (only a few eV difference). The calculations with hydrogen wave functions ([2]), however, differ by a factor of 2 to 3. From Table II it can also be seen that 82% of the observable effect of the electron screening comes from the electronic K-shell, 14% from the electronic L-shell and only small amounts from the higher electronic shells.

New experiments with an energy resolution of only some 10 eV indicate that the observed spectral lines might have a « natural broadening » ([14]) or an asymmetric line shape ([2]). This can be explained in a very simple way: It is evident that statistically the atoms will have different degrees of ionization which are expected to depend on the particular level occupied by the muon. Thus the electron screening is expected to have a statistical distribution of some 10 eV as it is shown in Table II. This effect can be even larger for the higher transitions. In fact the precise line shape can, therefore, give information on the probability distribution of the various degrees of ionization. This in turn may yield interesting information on the capture mechanism of the muon.

In our calculation we get automatically the electron binding energies as well. Since the muon stays in the 1s level for 10^{-6} seconds it might be possible to make interesting X-ray or optical-spectroscopy experiments on those atoms. The electron levels are expected to be very near to those of the element $Z-1$ because the muon screens the nuclear charge for the outer electrons.

To give an idea of this effect we have listed the binding energies of some inner electron shells in Table III for lead without and with the muon in the 1s level. For com-

([13]) D. LIBERMANN, J. T. WABER and D. T. CROMER: *Phys. Rev.*, **137**, A 27 (1965).
([14]) E. KANKELEIT: private communication.

TABLE III. – *The energy levels of some inner electron shells (in* Ryd) *for* Pb, Pb *with a muon in the* 1*s state and the* $Z-1$ *element, i.e.* Tl.

Level	Pb	Pb + 1*s* muon	Tl
$1s_{1/2}$	6455.71	6274.24	6273.32
$2s_{1/2}$	1153.81	1116.42	1116.17
$2p_{1/2}$	1109.02	1072.32	1072.27
$2p_{3/2}$	948.30	920.68	920.65
$3s_{1/2}$	276.84	266.40	266.32
$3d_{3/2}$	186.63	179.39	179.37
$4s_{1/2}$	62.54	59.38	59.33

parison the binding energies of the $Z-1$ element, *i.e.* tallium, are added. This Table indicates that the differences in the transition energies between electronic states in a muonic Pb atom and those in the Tl atom are very small. Nevertheless these differences might be observable.