In: Teaching and learning mathematics in context, Edited by Breiteig (etc.), 1993, Ellis Horwood Limited, Chichester, S. 3-14

1

Mathematical modelling in mathematics education and instruction

Werner Blum
Mathematics Department, Kassel University, Germany

SUMMARY

This paper aims at giving a concise survey of the present state-of-the-art of mathematical modelling in mathematics education and instruction. It will consist of four parts. In part 1, some basic concepts relevant to the topic will be clarified and, in particular, mathematical modelling will be defined in a broad, comprehensive sense. Part 2 will review arguments for the inclusion of modelling in mathematics teaching at schools and universities, and identify certain schools of thought within mathematics education. Part 3 will describe the rôle of modelling in present mathematics curricula and in everyday teaching practice. Some obstacles for mathematical modelling in the classroom will be analysed, as well as the opportunities and risks of computer usage. In part 4, selected materials and resources for teaching mathematical modelling, developed in the last few years in America, Australia and Europe, will be presented. The examples will demonstrate many promising directions of development.

1 WHAT IS MATHEMATICAL MODELLING?

There are as many definitions of mathematical modelling as there are authors writing about it – quite apart from the spelling with one 'l' or two – and the annual number of publications on modelling is increasing steadily. Let me mention some useful recent books in English that deal with the question of what modelling is: Blum, Niss and Huntley (1989), Clements (1989), Edwards and

Hamson (1989), Huntley and James (1990), Murthy, Page and Rodin (1990), Starfield, Smith and Bleloch (1990). Of course, older books such as Cross and Moscardini (1985) or Giordano and Weir (1985) are still useful, too, as well as the series of proceedings of the International Conferences on the Teaching of Modelling and Application (ICTMA): Berry *et al* (1984), Berry *et al* (1986, 1987), Blum *et al* (1989), Niss, Blum and Huntley (1991), de Lange *et al* (1993).

Now, as a basis for the following parts, I shall give some pragmatic working definitions which have been widely accepted in mathematics eduction in recent years (see the survey article by Blum and Niss, 1991). Let me quote the well-known simple model of *applied mathematical problem solving.*

The starting point is a *situation* in the real world, that means in the rest of the world outside mathematics. The situation normally has to be simplified, structured and made more precise by the problem solver, which leads to a *real model* of the situation. The real model is not – as one will often read – merely a simplified but true image of some part of an objective, pre-existing reality. Rather the step from situation to model also creates a piece of reality, dependent on the intentions and interests of the problem solver.

Then, if possible (if there is some mathematics in it), the real model is *mathematised*, that is translated into mathematics, resulting in a *mathematical model* of the original situation. Sometimes, different models of the same situation may be constructed.

The problem-solving process continues by choosing suitable methods and working within mathematics, through which certain *mathematical results* are obtained. These have to be retranslated into the real world – that is, to be *interpreted* in relation to the original situation. In doing so, the problem solver also validates the mathematical model. If discrepancies occur – which will often happen in reality, since there are so many potential pitfalls (see Murthy, Page and Rodin, 1990) – then the whole cycle has to start again.

Actually, all of the above is valid only for 'really real' situations. Sometimes – especially in school mathematics – the given situation is just an artificial dressing-up of some purely mathematical problem. Then the model-building process consists merely of an undressing of this *word problem*, and the problem-solving process stops after only one cycle. Nevertheless, such artificial problems may be of didactical value.

The term *application* of mathematics may be used for different parts of this cycle. A real-world situation may be called an application, using mathematics to investigate real situations may be denoted applying mathematics, or any manner of connecting the real world with mathematics can be seen as an application of mathematics. Equally, the term mathematical *modelling* may mean the process of model building, leading from a real situation to a mathematical model, or the whole applied problem-solving process, or again any manner of connecting the real world with mathematics. In recent years, the term *applications and modelling* (or vice versa) is frequently used as an all-embracing expression for the various interrelations just mentioned. In the following, in order to simplify matters, I will use the term (mathematical) *modelling* mostly in this comprehensive and extensive sense. This terminology is also justified by the fact that there is an international trend towards broadening the views on applications, models, mathematising, modelling, links between mathematics and other subjects, and so on. Thus the relations between these components have become stronger in the last ten years, both in research and in practice, as identified in Blum and Niss (1991).

2 WHAT IS MODELLING GOOD FOR IN MATHEMATICS TEACHING?

In the last 15 years there has been a worldwide trend towards modelling (in the broad sense) in mathematics education, at school and university levels. In particular, there has been a recent shift from applications to modelling, that is a trend towards emphasising the translation processes and not only working with ready-made models.

Today, modelling is favoured by almost everybody, as are problem solving, active learning and other activities of this nature (see, for example, the Cockcroft Report in the UK or the NCTM Recommendations in the USA). There are various *reasons* for favouring modelling in mathematics teaching. I would like to mention briefly four essential arguments, based mainly on general goals and aims for mathematics instruction (see Blum, 1991).

- *Pragmatic arguments.* Mathematics teaching is intended to help students to understand and to cope with real-world situations and problems. To that end, modelling is indispensable.
- *Formative arguments.* By being concerned with mathematics, students should – we hope – acquire general qualifications (such as the ability to tackle problems) or attitudes (such as openness towards new situations).

Modelling is one important way to develop these.

- *Cultural arguments.* Students should be taught mathematical topics as a source for reflection, or in order to generate a comprehensive and balanced picture of mathematics as a science and a part of human history and culture. Modelling is an essential feature of human intellectualism as well as of history and of actual practice, and can thus contribute towards promoting those aspects.
- *Psychological arguments.* Mathematical contents can be motivated or consolidated by suitable modelling examples, and these may contribute towards deeper understanding and longer retention of mathematical topics, or they may improve students' attitudes toward mathematics.

Let me put it even more generally. At all times and places there have been complaints about the lack of sense and the absence of meaning in mathematics learning and teaching. Students very often experience mathematics as a mechanical manipulation of meaningless symbols. Modelling can contribute towards giving more *meaning* to the learning and teaching of mathematics. If the formation of appropriate basic ideas and the development of meaning are essential aims of mathematics instruction, then modelling has to be an integral part.

The debate in mathematics education on modelling is not uniform. Different persons and groups emphasise different aims for mathematics teaching, and different arguments for the inclusion of modelling. Accordingly, it is possible to identify certain *schools of thought* within mathematics education. Kaiser-Messmer (1991) has discerned three of them:

- a so-called *pragmatic* school, broadly accepted in the English language area, where pragmatic aims are placed in the foreground;
- a so-called *scientific-humanistic* school, widely accepted in the Romance language area, where cultural aims are stressed;
- a so-called *integrated* school, largely found in the German language area.

However, since the eighties, there has been a trend in mathematics education all over the world towards widening the spectrum of aims and arguments, and thus to accept all arguments as important. In any case, no further advertising seems to be necessary, as there is a consensus of opinion in the mathematics education community that modelling has to be an integral part of mathematics instruction. Now, what is the actual situation in mathematics curricula and especially in school and university classrooms? Here, in my experience, a very different picture arises.

3 WHAT IS THE ROLE OF MATHEMATICAL MODELLING IN CURRICULA AND IN EVERYDAY TEACHING PRACTICE?

When considering modelling in mathematics *curricula*, there are differences with respect to

- the aims of the mathematics teaching and the corresponding rôle of modelling,
- the range of mathematical topics with modelling content,
- the areas that modelling examples are taken from (science, economy, everyday life, sports and so on),
- the kinds of examples (global – local, authentic – artificial, open – closed, relevant – irrelevant for students, and suchlike)
- the approaches to the organisation of mathematical and modelling curriculum components (separated, mixed, integrated).

Globally speaking, there is a clear world-wide trend towards including more modelling into mathematics curricula, also – though to a smaller extent – modelling in the strict sense. Some present-day secondary school curricula, for instance from the Netherlands or from Victoria in Australia, have included compulsory modelling components throughout. Furthermore, both the range of applied mathematical topics and of applicational areas have broadened – for example, more so-called discrete mathematics is included, at all levels from lower secondary to tertiary.

In everyday *teaching practice*, the amount of mathematical modelling work is also increasing. Particularly standard models – small, well-defined examples which are easy to survey and fit in well with the conventional curriculum – have entered the classroom. However, there is still a substantial gap between the forefront of research and development in mathematics education, on the one hand, and the mainstream of mathematics instruction, on the other hand. In most countries, modelling (in the broad and, even more so, in the strict sense) still plays only a minor rôle in everyday teaching practice at school and university. Formal calculations prevail, and applicational examples are treated, if at all, only as illustrations, and are not taken seriously as such. Even competent and committed teachers often do not follow all those well-founded recommendations and suggestions of mathematics educators. Why is this?

I think it is not due to ill-will or incompetence in teachers but to actual barriers and obstacles. Let me summarise briefly three of these.

- Obstacles from the point of view of *instruction and assessment*. Modelling requires a lot of time, and fits neither the regular mathematics syllabus (which is overloaded anyway) nor the regular school or university organisation (with 45 minute lessons and suchlike). Modelling is difficult to assess (see Niss, 1993), and what is not examined will not be taken seriously by students or by teachers.
- Obstacles from the *student's* point of view. Modelling makes mathematics lessons and examinations more demanding and less predictable. We have renewed evidence of this fact from our recent comparative empirical studies in England and Germany (see Blum *et al,* 1992).
- Obstacles from the *teacher's* point of view. Modelling makes teaching more demanding, because additional non-mathematical knowledge and qualifications are required, as well as the ability to manage open situation in the classroom. So the teacher's rôle is changed. Furthermore, teachers often do not know enough modelling examples suitable for instruction, and the ones they have at hand have to be brought up-to-date and have to be adapted for the class, for which a great deal of additional effort has to be invested.

One might think that the last point is a fourth obstacle, namely that there are not enough modelling examples and materials suitable for teaching. I am convinced that this is not true. For nearly every mathematical topic area there is a wealth of modelling examples of all kinds, in textbooks, case collections, teaching units, books or articles, all in principle accessible to teachers. I will name a few recent materials below.

There is a lot being done to overcome those obstacles, in curriculum development or in pre-service and in-service teacher education. According to my observations, the situation in the classroom is in the process of being improved. In countries all over the world, teachers in schools, colleges and universities are becoming more and more aware of deficiencies and trying to bring about change in the desired directions – towards a broader range of aims, towards more applications, towards more authentic examples, towards more open problems, and so on. Therefore there is a clear trend towards reducing the gap mentioned earlier, though with considerable differences between various countries.

I am convinced, though, that the barriers for modelling are very difficult to surmount. I see two things especially that need to be done. First, to initiate all kinds of teacher education activities, in order to supply as many teachers as possible with knowledge, abilities and, in particular, with attitudes to cope with the demands of teaching modelling – up to now, only very few reports on modelling experiences in teacher education exist. Second, to develop appropriate modes of assessment. Here we can learn a lot from the Dutch and the Australian materials and from their experiences, both good and bad. All these kinds of activities have to be accomplished essentially by teachers themselves.

There is some hope that the situation in mathematics classroom will improve especially through the increasing availability of powerful *computers*. Without doubt, new exciting opportunities have emerged for making contents accessible to learners, or for relieving mathematics teaching and learning of some tedious activities. This has obvious implications for mathematical modelling, too. In fact, there is a trend towards an extended use of computers in mathematics teaching. However, we should not expect too much. First, the obstacles just mentioned remain largely unaffected by the presence of computers; on the contrary, even more qualifications and higher-level abilities are demanded from students and teachers. Second, computers may entail many kinds of new problems and risks – for instance, students may try to replace necessary intellectual effort with modelling by mere button pressing. Third, there are and will be organisational problems with the accessibility of personal computers. I am convinced that nothing but powerful and cheap pocket computers will really affect mathematics teaching.

4 WHAT MATERIALS ARE AVAILABLE FOR TEACHING MATHEMATICAL MODELLING?

As to older resources, I refer to the surveys from the International Congresses on Mathematical Education (ICME): Pollak (1979), Bell (1983), Niss (1987), Blum and Niss (1991).

In the last few years, many interesting new materials have been developed. Let me present a small, subjective selection, concentrating on the secondary-school level.

From Australia

- Two booklets by Carr and Galbraith (1987, 1991) from the project PAM, with detailed examples and teaching units for the lower-secondary level, covering a broad range of extra- and intra-mathematical subjects.
- Two volumes by Lovitt and Clarke (1988), an extensive collection of detailed examples for the lower-secondary level, oriented towards students' activities.

- Two books by Lowe (1988, 1991), with numerous detailed examples for grades 7–12, covering a broad range of subjects, mostly referring to computers as a tool.
- In Australia, there are also many efforts towards new assessment modes for modelling-oriented curricula (see, for example, Clatworthy and Galbraith, 1991).

From the Netherlands

- The Freudenthal Institute (before 1991 known as OW&OC) at Utrecht University has developed lots of materials, including textbooks, for all grades in school: primary (the Wiskobas project by Treffers *et al*), upper-secondary (the Hewet project by de Lange *et al)*, and more recently also lower-secondary. Many materials have also been published in English, especially in connection with the Madison project (see de Lange, 1992). All materials are structured according to mathematical topics, and the modelling examples are intended to support the learning of the mathematics.

From the UK

- Several booklets and materials from the Numeracy Through Problem Solving project, developed in the Shell Centre for Mathematical Education at Nottingham University (involving Burkhardt *et al*).
- The Enterprising Mathematics Course from the Centre for Innovation in Mathematics Teaching at Exeter University (by Burghes *et al*).
  Both the Shell and the CIMT materials are written for the lower-secondary level, and both are structured according to problems, not to mathematics, and aim at stimulating students' activities.
- A series of booklets for the A-level from the Spode Group (1992), with detailed examples for direct use in the classroom.
- I also refer once more to the books mentioned at the beginning of this paper, all containing many case studies, mostly for the tertiary level.

From the USA

- Two series of books with numerous modelling examples from the Consortium for Mathematics and its Applications (involving Garfunkel, Aragon, Malkevitch *et al*) are the HIMAP Modules (1985-92) for the secondary level and the UMAP Modules (1981-92) for tertiary and upper-secondary levels.
- The book by Garfunkel and Steen (1991), an introduction to recent mathematical applications in practice, especially in connection with computers, suitable for secondary and tertiary levels, with some mathematical topics as yet outside present syllabi.

- A series of application-oriented textbooks (1989-92) from the University of Chicago School Mathematics project (involving Usiskin, Bell *et al*), covering arithmetic, algebra, geometry, statistics, precalculus and discrete mathematics.
- From NCTM an introduction to college mathematics (1988), developed by the North Carolina School of Science and Mathematics (Teague *et al*), and a booklet by Swetz and Hartzler (1991) with modelling examples for the secondary level, oriented towards students' activities.

From Germany

- The project MUED (Mathematik-Unterrichtseinheiten-Datei involving Böer, Volk *et al*) has developed several detailed global teaching units aiming at students' abilities to act competently in real-life situations. Two examples for the lower-secondary level are Böer (1990) and Jannack (1992).
- Several projects at the university level deal with real uses of mathematics in industry, and their utilisation in the training of future mathematicians or mathematics teachers. An example is the case studies in Knauer (1992).

Many more references to literature for the secondary-school level in German, English and French, are contained in the extensive bibliography by Kaiser-Messmer, Blum and Schober (1982/1992).

Altogether, these examples demonstrate many promising *directions of development* from various parts of the world.

- Application-oriented mathematics curricula on a broad scale in the Netherlands.
- Case collections for new modelling-oriented mathematics curricula, including assessment, in Australia.
- Examples of activity-oriented global-problem sequences in England.
- Materials for a new orientation of mathematics curricula in the computer age in the USA.
- Application-oriented teaching units as alternatives to conventional mathematics instruction in Germany.

In addition to these, there are many extremely interesting activities in several other countries that I could not present in this paper. Of course, many past and present noteworthy activities in the field of learning and teaching mathematical modelling are unknown to the public simply because of language problems or since only non-official publications exist. However that may be, I apologise for any omissions.

So, looking ahead, in my view we could and should be rather optimistic, despite a number of still unsolved or perhaps even unsolvable problems. I trust that this will not only be my view – optimism is probably a necessary universal attitude for mathematics teachers and educators.

REFERENCES

Bell, M (1983). Materials available worldwide for teaching applications of mathematics at the school level. In Zweng, M *et al* (ed), *Proceedings of the Fourth International Congress on Mathematical Education.* Boston: Birkhäuser, 252-267.

Berry, J *et al* (ed) (1984). *Teaching and Applying Mathematical Modelling.* Chichester: Ellis Horwood.

Berry, J *et al* (ed) (1986). *Mathematical Modelling Methodology, Models and Micros.* Chichester: Ellis Horwood.

Berry, J *et al* (ed) (1987). *Mathematical Modelling Courses.* Chichester: Ellis Horwood.

Blum, W (1991). Applications and modelling in mathematics teaching - a review of arguments and instructional aspects. In *Niss, M, Blum, W and Huntley, I* (ed), loc. cit., 10-29.

Blum, W *et al* (ed) (1989). *Applications and Modelling in Learning and Teaching Mathematics.* Chichester: Ellis Horwood.

Blum, W *et al* (1992). Teaching and learning of mathematics and its applications: first results from a comparative empirical study in England and Germany. In *Teaching Mathematics and its Applications,* **11**, 112-123.

Blum, W and Niss, M (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. In *Educational Studies in Mathematics,* **22**, 37-68.

Blum, W, Niss, M and Huntley, I (ed) (1989). *Modelling, Applications and Applied Problem Solving - Teaching Mathematics in a Real Context.* Chichester: Ellis Horwood.

Böer, H (1990). *Das Projekt Wasser.* Mühlheim: Schulpraxis.

Carr, A and Galbraith, P (1987/1991). *Practical Applications of Mathematics.* Vol 1/2. Churchill: PAM.

Clatworthy, N and Galbraith, P (1991). Mathematical modelling in senior school mathematics: implementing an innovation. In *Teaching Mathematics and its Applications,* **10**, 6-22.

Clements, R (1989). *Mathematical Modelling - A Case Study Approach.* Cambridge: Cambridge University Press.

COMAP (ed) (1985-92). *HIMAP Modules 1-19.* Arlington: COMAP.

COMAP (ed) (1981-92). *UMAP Modules - Tools for Teaching.* Arlington: COMAP

Cross, M and Moscardini, A (1985). *Learning the Art of Mathematical Modelling.* Chichester: Ellis Horwood.

Edwards, D and Hamson, M (1989). *Guide to Mathematical Modelling.* London: Macmillan.

Garfunkel, S and Steen, L (ed) (1991). *For All Practical Purposes.* New York: Freeman.

Giordano, F and Weir, M (1985). *A First Course in Mathematical Modelling.* Monterey: Brooks/Cole.

Hobbs, D (ed) (1992, 1993). *Enterprising Mathematics Course.* Oxford: Heinemann.

Huntley, I and James, G (ed) 1990. *Mathematical Modelling - A Source Book of Case Studies.* Oxford: Oxford University Press.

Jannack, W (1992). *Schalten mit Köpfchen.* Mühlheim: Schulpraxis.

Kaiser-Messmer, G (1991). Application-orientated mathematics teaching: a survey of the theoretical debate. In *Niss, M, Blum, W and Huntley , I (ed),* loc.cit, 83–92.

Kaiser-Messmer, G, Blum, W and Schober, M (1982, 1992). *Dokumentation ausgewählter Literatur zum anwendungsorientierten Mathematikunterricht.* Vol 1/2. Fachinformationszentrum, Karlsruhe.

Knauer, U (1992). *Mathematische Modellierung.* Braunschweig: Vieweg.

de Lange, J (1992). *Meaningful Math.* 3 Volumes: Matrices; Flying Through Math; Data Visualization (with Verhaage, H). Scotts Valley: Sunburst.

de Lange, J *et al* (ed) (1993). *Innovation in Maths Education by Modelling and Applications.* Chichester: Ellis Horwood.

Lovitt, C and Clarke, D (1988). *MCTP Professional Development Package:* Activity Bank. Vol 1/2. Canberra: Curriculum Development Centre.

Lowe, I (1988/1991). *Mathematics at Work: Modelling your World.* Vol 1/2. Canberra: Australian Academy of Science.

Murthy, D, Page, N and Rodin, E (1990). *Mathematical Modelling - A Tool for Problem Solving in Engineering, Physical, Biological and Social Sciences.* Oxford: Oxford University Press.

Niss, M (1987). Applications and modelling in the mathematics curriculum - state and trends. In *International Journal for Mathematical Education in Science and Technology* **18,** 487-505.

Niss, M (1993). Assessment of mathematical modelling and applications in mathematics teaching. In de Lange, J *et al (ed),* loc. cit, 41-52.

Niss, M, Blum, W and Huntley, I (ed) (1991). *Teaching of Mathematical Modelling and Applications.* Chichester: Ellis Horwood.

North Carolina School of Science and Mathematics (1988). *Introduction to College Mathematics.* 2 Volumes: Matrices; Data Analysis. Reston: National Council of Teachers of Mathematics.

Pollak, H (1979). The interaction between mathematics and other school subjects. In UNESCO (ed), *New Trends in Mathematics Teaching IV.* Paris: UNESCO, 232–248.

Spode Group (1992). *A-Level Mathematics Support Series.* Vol 1–10. Cranfield: Cranfield Press.

Starfield, A, Smith, K and Bleloch, A (1990). *How to Model it: Problem Solving for the Computer Age.* New York: McGraw Hill.

Swetz, F and Hartzler, J (ed) (1991). *Mathematical Modelling in the Secondary School Curriculum.* Reston: National Council of Teachers of Mathematics.

University of Chicago School Mathematics Project (Usiskin, Z *et al*) (1989-92). *6 Volumes: Transition Mathematics; Algebra; Geometry; Advanced Algebra; Functions, Statistics and Trigonometry; Precalculus and Discrete Mathematics.* Glenview: Scott, Foresman & Co.