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British/German Comparative Project: Some Preliminary Results

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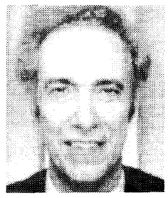


1. Introduction

THE UNIVERSITIES of Exeter (Centre for Innovation in Mathematics Teaching) and Kassel (Mathematics Department) have been working closely together over the past few years as they have a common interest in encouraging the teaching of mathematics through its applications. This early work highlighted the fact that there were significant differences between German and British methods of teaching mathematics, let alone its applications. A full account of this preliminary study* is given by Blum *et al.*[1] The German style is characterised by

(1) the teacher clearly dictating the interaction and path of the lesson;
(2) class discussion being a very important part of the lesson;
(3) the teacher having very strict control over homework;
(4) emphasis being placed on correct, precise mathematical language at all times.

In contrast, the English style is characterised by

(1) less formal teacher-led discussion, with the emphasis on individual pupil-led work;
(2) class discussion between teachers and individual pupils as the main teaching form, in which difficulties were sorted out individually;
(3) greater emphasis on self-paced work schemes;
(4) less emphasis on correct mathematical expressions.

Whilst there is much variation in the classroom styles within the two countries, it is clear from our preliminary study that there are, on the whole, significant differences in the general approaches.

* Funded by the British Council

2. Methodology

Comparisons between countries have been given some prominence recently, including the work of Prais and Wagner,[2] which related to Germany and the United Kingdom. Many of these studies, however, have used inaccurate methods of comparison. In order to be sure that we are comparing groups of similar mathematical ability, we have now developed a *mathematical potential test* from which we will be able to select students of similar mathematical ability. We plan to follow their mathematical progress over a number of years in order to determine what factors are instrumental in enhancing progress in mathematics.

The mathematical potential test has now been piloted, revised, and tested several times and this preliminary testing has in itself led to some interesting results. The revised version of this test is given in the Appendix.

This test, and its German equivalent, has been used in a number of contrasting schools in both countries. In Germany, the sample size was 302, roughly divided in proportion to the numbers of pupils in the three main types of schools in Germany, namely,

Gymnasium	45%
Realschule	30%
Hauptschule	25%

In Britain, the sample size was 540, all of whom were in comprehensive schools, but in areas where a significant proportion of high-ability children would have been attending private schools. The test was time-limited and given to pupils aged 12–14, although the average age of the British sample was a little less than the German average.

The test was designed to be relatively "content" free, since it was designed as a test of potential, rather than mathematical knowledge. The broad areas covered by the test are:

Simple calculations with numbers	(2, 5)
Recognising structure of number sequences	(1, 17, 19, 22)
Applied algebraic calculations	(8, 10, 13, 24)
Handling proportions	(11, 15, 20, 26)
Puzzles on number symbols	(28)
Plane figures	(3, 12, 16)
Spatial ability	(7, 21)
Interpreting graphs in context	(6, 23)
Concept of probability	(9)
Combinatorial reasoning	(14, 27)
Recognising structures	(4, 18)
Logical reasoning	(25)

There was a time limit of 40 minutes for the test, and calculators were not allowed. A summary of the full results is given in Table I, and the χ^2 test has been used to test for significant differences.

Note that Question 9 was inadvertently changed in translation, being made considerably easier in the German version, so the results for this question have not been included in the analysis.

3. Interpretation of results

The most surprising result, given that the samples were not chosen deliberately to be comparable, was that in almost half the questions (13 out of 27), there was no significant difference between the performances of British and German pupils. For those questions in which there were significant differences in performance, 12 were in favour of German pupils and 2 in favour of British pupils. Particular difference can be partially explained by the different emphasis put on the teaching of topics in the two countries, whilst overall differences can also be attributed to the creaming off of many high-ability pupils in the United Kingdom into the private sector of education.

Detailed differences and interpretations are given below.

(1) Arithmetic/algebraic topics

German pupils performed better in all questions on calculations with *number* and *proportion.*

For the questions on *applied algebraic calculations*, a somewhat different picture emerges. The German pupils obtained significantly better results in two questions, whereas the British pupils obtained better results (not significant) in two other questions that required more advanced algebraic or (more appropriate) trial-and-error approaches. One possible explanation for the better results of the British pupils in these two questions might be that they were more used to trial-and-error methods, such methods being common in the investigational work so often seen in British schools.

In the questions relating to the *structure of number sequences*, the German pupils obtained better results in three out of four questions (significant in two questions).

The question in which the British pupils obtained better results used *square numbers*, which were not known to the pupils of one of the participating German schools.

In the only question on *puzzles on number symbols*, the German pupils also obtained significantly better results.

These results can be readily explained by the high importance that is given to these topics in German mathematics lessons and the teaching time that is devoted to them. German mathematics teaching may be characterised in part by its orientation towards an accurate performance of computational algorithms, especially those of simple calculations without calculators, of proportions, percentages and fractions. This is in contrast to most British mathematics teaching, which gives relatively little importance to the practice of computational algorithms and very often emphasises investigative work, the effects of which are much more difficult to test.

(2) Geometrical topics (including graphs)

Here the British pupils obtained significantly better results.

In the questions on *two-dimensional shapes*, the British pupils obtained significantly better results in two out of three questions (one at the 2.5% level, one at the 5% level). In one of these questions—asking for shapes in a grid—the biggest difference in favour of the British pupils emerged. A plausible explanation for this difference is that shapes in a grid are usually not treated in German mathematics lessons and were therefore totally unfamiliar to the German pupils.

With *three-dimensional shapes*, in one question the German pupils obtained better results, whilst in the other questions the British pupils were better (but not significantly).

In the two tasks on *contextual graphs*, the British pupils obtained better results, neither significant at the 5% level. This again can be explained by the lower emphasis given to studying and interpreting

Table I
Results of the Mathematical Potential Test

Question Number	Britain (n = 540) No. correct	%	Germany (n = 302) No. correct	%	χ^2 G > B	B > G	Significance Level
1	531	98	300	99	1.515		
2	426	79	275	91	20.578		$p < .01$
3	427	79	217	72		5.613	$p < .025$
4	367	68	214	71	0.760		
5	486	90	299	99	24.890		$p < 0.1$
6	226	42	106	35		3.698	
7	349	65	210	70	2.090		
8	178	33	128	42	7.431		$p < 0.1$
9	347	64	271	90	**Not applicable**		
10	340	63	227	75	13.113		$p < 0.1$
11	178	33	152	50	24.515		$p < 0.1$
12	209	39	68	23		22.989	$p < 0.1$
13	353	65	179	59		3.100	
14	355	66	212	70	1.750		
15	140	26	131	43	27.026		$p < 0.1$
16	388	72	235	78	3.422		
17	121	22	56	19		1.186	
18	224	42	110	36		2.070	
19	240	44	193	64	29.371		$p < 0.1$
20	93	17	79	26	9.516		$p < 0.1$
21	214	40	116	38		0.121	
22	422	78	256	85	5.412		$p < 0.025$
23	181	34	98	32		0.100	
24	72	13	27	9		3.602	
25	158	29	151	50	35.685		$p < 0.1$
26	117	22	85	28	4.459		$p < 0.05$
27	24	4	21	7	2.411		
28	87	16	95	31	26.920		$p < 0.1$

	Britain	Germany
Mean (excluding Q. 9)	12.73	14.05
Standard deviation	4.96	4.82

Questions with differences in favour of the German pupils:

1, **2**, 4, **5**, 7, **8**, **10**, **11**, 14, **15**, 16, **19**, **20**, 22 (2.5% level), **25**, 26 (5% level), 27, **28**

Questions with differences in favour of the English pupils:

3 (2.5% level), 6, **12**, 13, 17, 18, 21, 23, 24

N.B. (**Bold** : significant differences at the 1% level) graphs in German mathematics lessons compared to British lessons.

(3) Combinations/probability/logic topics

Here the German pupils obtained better results in four out of five questions, but only one difference was significant (at the 1% level).

In one ambitious question (27) testing *general combinatorial skills*, nearly all British and German students failed.

In the two questions on *symbolic structures*, the results were not uniform: in one question the German pupils obtained better results and in the other the British pupils did better. (However, neither was significant.)

In the *logic* question the German pupils obtained significantly better results; in fact this gave the most significant difference (measured with χ^2) in favour of the German pupils in the whole test. Although logical topics are not usually explicitly included in German syllabuses, mathematics teaching, at least for the Gymnasium, places great emphasis on logical reasoning and reflection.

4. Summary

Our detailed analysis suggests that the test results are highly dependent on the mathematical instruction given to the pupils in the samples, despite the fact that we were concerned to find the potential ability of pupils in mathematics, rather than their actual level of performance. Generalisation concerning the differences between German and British pupils in mathematics must be treated carefully—there is often an obvious reason why differences show up. In particular, our results seem to show that (as we had hoped) the differences in mathematical potential between the pupils in the two countries are not particularly significant, and that differences that do exist can be readily explained by the emphasis put on these topics in mathematics teaching in each country.

We hope that our *potential test* will provide a valid yardstick from which to base comparisons. For the reasons elaborated above, we have decided to eliminate Questions 11 and 15 and replace Question 25 from the agreed version to be used in the next phase of our project. In this phase, we will be following the progress of a number of cohorts of children (aged about 11 and 14 years) for three years, regularly testing for attainment on particular mathematical topics, as well as assessing their mathematical potential. In this way, we will be able to make comparisons concerning the progress of children who start with similar potential. We will seek to correlate this progress with some possible influencing factors, such as schemes of work, size of class, style of teaching and type of school with a view to making recommendations about how to improve mathematics teaching in order to enable pupils to realise their mathematical potential.

References

1. Blum, W., Burghes, D., Green, N. and Kaiser-Messmer, G., *Teaching Mathematics and its Applications*, 1992, **3**, No. 3, 112–118.
2. Prais, S. and Wagner, K., *National Institute Economic Review*, May 1985, 53–76.

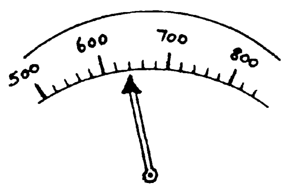
Appendix
The Mathematical Potential Test

Answer all questions in the spaces provided.
A calculator may not be used, but pen and paper calculations are allowed.

1 What is the next number in this sequence?

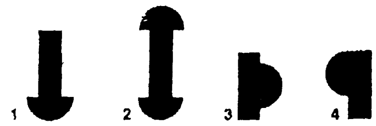
25 20 15 10 ... □

2 What number is the arrow pointing to?

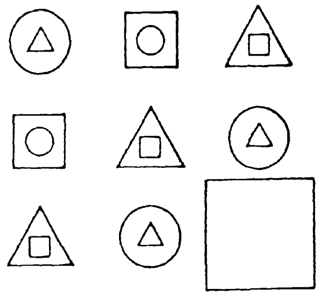


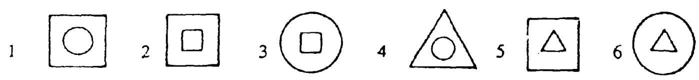
□

3 Give the number(s) of any of the shapes shown below which can be made from the two pieces given (each piece should only be used once) □

Pieces given	
Combined shapes	

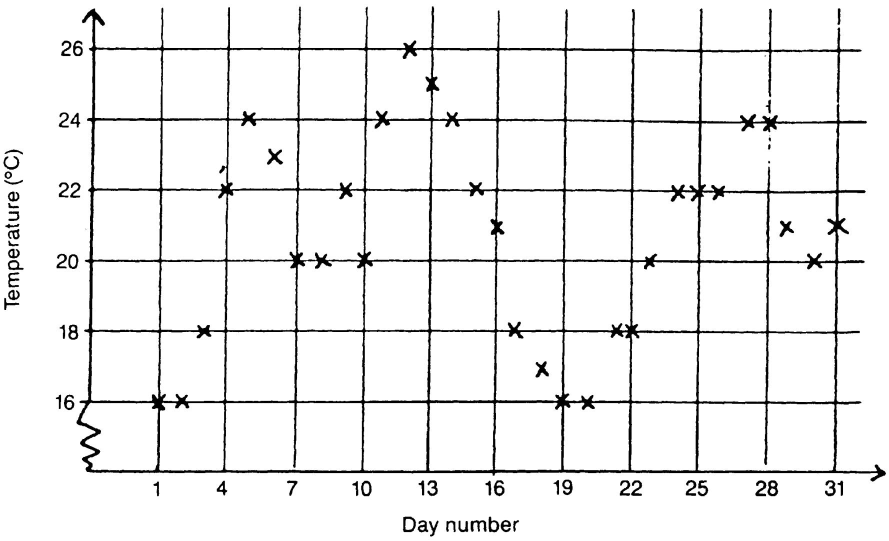
4 Give the number of the shape from those below which completes the pattern. □



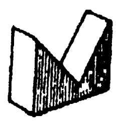


5 A woman has £100. She earns £40 more and spends £70. How much has she got now? □

6 The diagram shows the highest temperature each day during the month of July at a holiday resort. On how many days was the highest temperature at least 20°C? □



7 How many flat surfaces has this solid? □



8 A gardener is planting trays of seeds. She has 20 trays to do, and it takes her 10 minutes to plant each tray. She begins her work at half-past-seven in the morning. At what time will she have completed three-quarters of the trays? □

9 There are four bags containing black and white counters. □

Bag A: 12 black and 4 white
Bag B: 20 black and 20 white
Bag C: 20 black and 10 white
Bag D: 12 black and 6 white

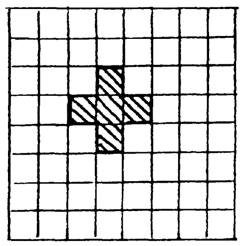
You have to take out one counter from a bag (with your eyes closed). Which bag gives you the best change of getting a black counter?

10 I think of a number. I then double it and take away 17. The answer is 45. What was the number? □

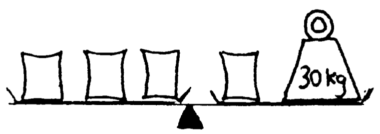
11 Twenty oranges cost £4.20. How much should twelve oranges cost? □

12 Draw as many crosses as possible on the grid so that:

- they are the same size as the one shown
- they do not overlap (they may touch).



13 Each sack weighs the same, and the scales balance. What does one sack weigh? □

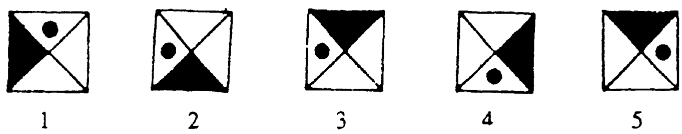


14 The five letters A, B, C, D, E are to be put into a 5 × 5 grid. Each row and each column must contain each letter. The first three rows are given. Complete the grid with two more rows.

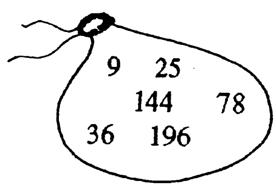
A	B	C	D	E
C	A	D	E	B
B	C	E	A	D

15 There are 1200 fish in a pond. This number increases by 15 fish each year for every 100 fish present at the beginning of the year. How many fish will be in the pond in a year's time? □

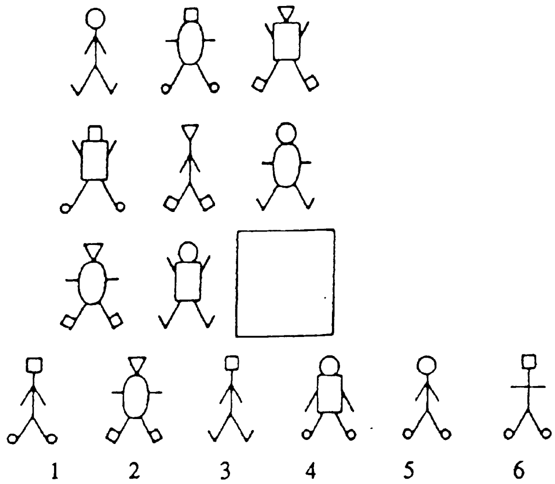
16 Which is the odd one out? □



17 Which number does not belong in this bag? □



18 Give the number of the shape from those below which completes the pattern? ☐

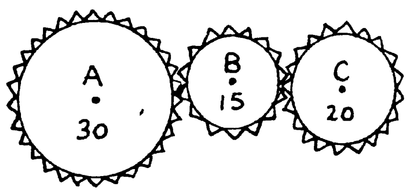


19 What are the next two numbers in the sequence?

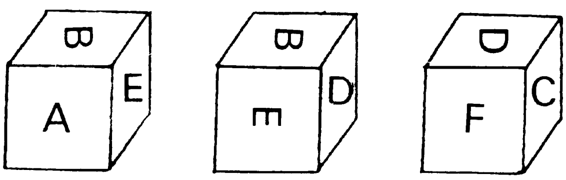
6 9 18 21 42 45 ... ...

... ...

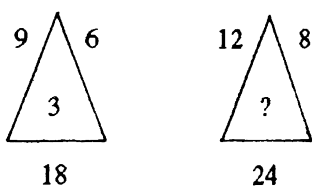
20 The diagram shows three cogs working together. The number of teeth is shown at the centre of each cog. If cog A makes six revolutions, how many will cog C make? ☐



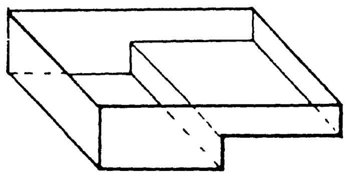
21 Here are three views of the same cube. On the faces of the cube are the letters A, B, C, D, E, and F. Which letter is on the face opposite to the one with the letter E? ☐

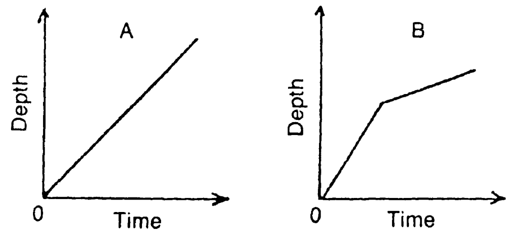


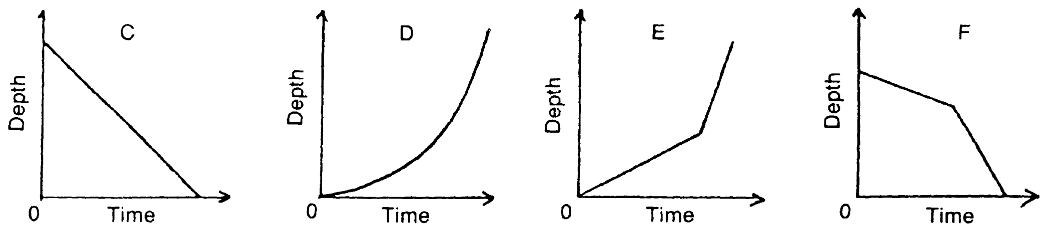
22 What is the missing number? ☐



23 This swimming-pool is filled with water at a constant rate. Which graph below best shows the increase in the depth of the water with the passing of time?







24 Tom, Dick and Harry have a sum of £575 to be shared between them. They agree to divide it so that Tom gets £19 more than Dick, and Dick gets £17 more than Harry. How much does Tom get?

25 Three children, whose first names are Ann, Brian and Carol live in the same road. Their ages are 7, 9 and 10, and their family names are Smith, Jones and Patel. If the girl with family name Smith is three years older than Carol, and the child with family name Patel is 9 years old, what is the family name and age of Brian?

....

26 A submarine has food to last its crew of 25 men for 6 months. How long would the food last a crew of 60 men?

27 There are 5 different kinds of sweets in a bag. What is the least number you must take from the bag to make sure that you get at least 3 of the same kind?

28 Each symbol in the multiplication stands for a different whole number. Find the value of each symbol.

■ ● × ● = ▲ ■ ●

■
●
▲

David Burghes is currently professor of education at Exeter University, with responsibility for mathematics, science and technology. He is also director of the Centre for Innovation in Mathematics Teaching, and his main crusade is to make mathematics more enjoyable and accessible for everyone. Before moving to Exeter he spent five years working at Cranfield, which strongly influenced his views on mathematics and education. He has published books on mathematics for both schools and higher education, loves travelling, and reads the British Rail timetables for relaxation.

Gabriele Kaiser-Messmer graduated from the University of Kassel with an MEd in Mathematics and Humanities. After teaching in schools for a number of years, she has spent the past ten years working on research projects in mathematics education. She completed her PhD in 1986 and recently has been working on comparisons of different international approaches to the teaching of mathematics. She has published many articles and given talks at a number of recent international conferences.

Nigel Green graduated from Nottingham University and has taught in comprehensive schools in Cheshire and Bradford; he is currently Head of Maths at Evesham High School. His wife is German and they have two children; he spent three years working in finance in Germany. He is a member of the Spode Group and is particularly interested in applications of mathematics.

Werner Blum has been Professor of Mathematics and Mathematics Education at Kassel University (Germany) since 1975. He was a Visiting Professor at Dortmund University in 1983/84, at Linz University (Austria) in 1985 and 1989, and several times at King Mongkut's Institute of Technology, Bangkok (Thailand). He received a Diploma (MSc) in Mathematics 1969 and a Dr. rer. nat. (PhD) also in Mathematics, both from Karlsruhe University. He has written and edited several books and numerous articles and research papers on mathematics education. The focal points of his research and development work are applications and modelling in mathematics teaching, mathematics in vocational education, and the teaching of calculus. He has been an enthusiastic football player since he was three years old, and also likes playing tennis and table tennis.

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