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Mathematics teaching in technical and vocational colleges - Professional training versus general education

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1. Vocational education in the Federal Republic of Germany

Training and education in technical and vocational colleges in the Federal Republic of Germany (FRG) can be divided into two main categories: There is part-time vocational training and education for the vast majority of young people in technical and vocational education (about 40% of the population aged 16 to 18), and full-time vocational (classroom-type) education for a minority (about 15%; for details cf. Weidig in this volume and Strässer 1982).

The *part-time vocational training and education* normally consists of classroom-type schooling one or two days per week complemented by three or four days of vocational training in companies (the West German "dual system" of vocational training; cf. the glossary in this volume). This type of initial vocational training normally lasts three years and offers a vocational certificate as a qualified worker (*Facharbeiter*) to the successful student. In the classroom part of the training, in the *Berufsschule* (cf. glossary), vocational arithmetics (*Fachrechnen*) is taught for two or three hours per week with the aim of giving a numerical foundation and interpretation of vocational phenomena, underpinning vocational knowledge with an arithmetical analysis ("zahlenmäßige Deutung und Durchdringung von beruflichen Erscheinungen"; see Wolff 1958; Strässer 1982).

Full-time vocational education takes place under a variety of organisational forms with a variety of aims and places in the student's career . There is one major distinction in full-time vocational education, namely the line between prevocational training after general education and "higher" vocational training. Prevocational training and education lasts for one or two years, normally preparing the student for a whole range of related professions (such as commerce, metalwork, or social vocations, e.g., in the Berufsfachschule, cf. glossary, or Berufsaufbauschule) and usually or legally offering the possibility to shorten the time spent in the dual system mentioned above. Higher professional training prepares students to upgrade their professional qualifications (e.g., in the Technikerschule) or to enter an academic career (e.g., in the Fachoberschule or Berufliches Gymnasium, cf. glossary). Mathematics is a subject in all these types of colleges and has the dual goal of repeating the mathematics of the preceding general education and giving the student an introduction to higher mathematics (e.g., calculus) to ease the transition to university education.

2. Mathematics teaching in vocational and technical colleges - An overview

In general as in vocational and technical education, the teaching of mathematics can have a broad variety of aims that are related to the overall aims of education and training (cf. Blum 1990):

1. *Pragmatic aims:* Mathematics teaching is intended to help students to describe and understand relevant aspects of extra-mathematical areas and situations and to cope better with them. Such situations may originate from:
a. present or future fields of study or profession, or from other vocational subjects at school, or
b. daily life and the world around us, or from other general subjects at school.

2. *Formative aims:* By being concerned with mathematics, students should:
a. acquire general qualifications; for instance, the ability to argue, to solve problems, or to translate between the real world and mathematics, as well as attitudes such as being comfortable when facing new problem situations and being willing to make intellectual efforts, and
b. obtain pleasure and enjoy their activities.

3. *Cultural aims:* Students should be taught mathematical topics:
a. as a source of philosophical and epistemological reflection, including individual and human self-reflection,
b. to generate a balanced, comprehensive picture of mathematics as a science and part of human history and culture, including a critical appreciation of actual uses (and misuses) of mathematics in society, and
c. to produce knowledge, skills, and abilities related to special mathematical themes, and to create a "meta-knowledge" of mathematics.

At a first glance, only Aim 1a seems to be vocational in a narrow sense, while all others seem to belong to general education. However, a subsequent analysis of the use of mathematics in the workplace shows a more differentiated picture. Such analyses turn out to be rather difficult for methodological reasons (cf. Bardy et al. 1985, pp.37-48; Strässer 1987); detailed studies should be undertaken by researchers competent in mathematics *and* in the vocational field in question and need to take into account a large number of variables (like the vocational domain, the size, technological standard and organisational standard of the company). In Germany, no such studies exist at present (and the rare studies done in other countries can be criticised on the grounds mentioned above). In addition, the needs of the workplace obviously change over time. There are also good indications that even specific mathematical qualifications can only be helpful and be used in the workplace if transfer-qualifications (as identified in Aim 2a) are at hand.

Information on the use of mathematics in the workplace shows the importance of elementary calculations with numbers and technical magnitudes (including percent and the rule of three or proportion) and the use of tables and diagrams. For technical vocations, basic algebra and geometry are widely used. At present, the growing use of computers and related organisational changes seem to be inducing a growing need for geometry in domains like metalwork, electricity, and architecture (especially in technical drawing or CAD and with CNC-machines; for details, see Bardy et al. 1988, pp. 151-170). In business and administration some basic ideas from statistics seem to be relevant.

2.1 Teaching and learning mathematics in part-time vocational colleges

In West German part-time vocational colleges, mathematics teaching aims at helping students describe, understand, and master special vocational situations and subjects. It concentrates on elementary calculations with numbers and technical magnitudes and on the use of tables and diagrams. For technical vocations, basic algebra (such as simple equations, formulas, and functions) is taught. Two - and (especially for metalwork and architecture:) three-dimensional-geometry are normally taught to the students or apprentices in specific technical drawing courses. Business and administration courses generally concentrate on a large variety of calculations of interest (like discount, bank drafts, bills, credit and loans, insurance). Some basic ideas from statistics play a specific role in the insurance business (cf. Blum 1979, Bardy et al. 1985, pp. 49-63). The majority of these topics also belong to the junior secondary curriculum and should simply be repeated in vocational colleges. Nevertheless, these topics have to be treated again in vocational education for a number of reasons: apart from the time lag between general and vocational education, general education seems to specifically neglect the use of technical or physical formulas as well as three-dimensional geometry.

In vocational colleges, these topics are normally taught in close relation to vocational and professional knowledge, and nearly no disciplinary mathematical structure is followed. Mathematics is taught and used as a mere tool and by-product of vocational knowledge. With an orientation reduced to Aim 1a, the teaching *method* normally follows three steps: the topic is introduced (or repeated) with the help of some model examples ("problems"). The solution process is condensed to hints and algorithms (often put into formal instructions like "keep in mind that ..."). Finally, the algorithm is inculcated by solving a number of schematised problems having the same structure (cf. Bardy et al. 1985, pp. 64-71). Reasons for the algorithms are normally not given. The solution process is reduced to the application of recipes to vocational and professional problems. There is nearly no discussion of strategies to link vocational and professional situations to mathematical concepts. As a consequence, differences in the use of mathematical concepts creep in with no legitimisation (e.g., the widespread use of algebraic formulas in technical vocations as opposed to the relative neglect of this topic in business and social vocations).

Apart from the dominance of Aim 1a in part-time vocational colleges, there are additional reasons for the teaching style described above. Teachers try to adjust their teaching style to the motives and qualifications they expect from their students and they think students or apprentices in part-time vocational colleges are not as interested in theory and intellectually oriented as university-bound students of the same age. Standardised final examinations (from extra-college bodies) also play a role in the definition of topics and teaching style. On the other hand, detailed analyses of the final mathematics examinations clearly show that the most important source of errors and mistakes are those coming from difficulties with finding the appropriate mathematical model for the problem to be solved. This is a clear indication of the fundamental role of Aim 2a, especially the ability to translate between the real world and mathematics, thus playing down the role of recipes and algorithms for tomorrow's qualified worker. In a broader perspective, one could ask whether aims from general education should be pursued more in vocational education than is the case today.

2.2 Teaching and learning mathematics in full-time vocational and technical colleges

The mathematics teaching in West German full-time vocational and technical colleges often imitates the mathematics teaching of comparable general education (especially the *Gymnasium*). "General qualifications" (especially the development of logical thinking, cf. Aim 2a) are at the heart of the curriculum, as well as special mathematical topics (e.g., elementary algebra and calculus, cf. Aim 3c) and the analyses of extra-mathematical situations from the present or future professional fields of prevocational education and future fields of study for higher vocational education (cf. Aim 1a). The most important *topics* in *Berufsfachschule* and *Berufsaufbauschule* are elementary algebra (for details see Bardy et al. 1988, pp.171-210) and - predominantly in technical colleges - geometry (pp.211-228). *Fachoberschulen* and *Berufliche Gymnasien* concentrate on calculus (pp. 229-254), analytic geometry and linear algebra (especially in technical domains) and statistics (for nontechnical colleges). Often, these colleges have to treat junior secondary topics (like solving equations) extensively in order to fulfil the demands for manipulative qualifications set by subsequent tertiary institutions (such as *Fachhochschulen* for future engineers or economists).

The dominant *teaching method* can be described as traditional solving of textbook problems ("Aufgabendidaktik" as described in Lenné 1975). Knowledge of how to cope with certain classes of textbook problems (e.g., analysis of graphs of functions in calculus) is presented in a teacher-centered classroom interaction and then practiced with numerous problems of the same type. Thus, teaching and learning are again centered on schemas and algorithms. Introducing concepts, argumentation and proofs, and epistemological reflection play only a minor role, if any. Realistic applications (in terms of translating reality into mathematics -mathematizing- and interpreting mathematical results in realistic situations) normally are not treated.

The reasons for this type of mathematics teaching in full-time vocational and technical colleges are partly the same as in part-time colleges: competencies ascribed to students as well as traditional types of examinations play an important role. Nevertheless, it is questionable whether this teaching is really appropriate for the students' careers and personalities. A broader scope of aims - especially by integrating more vocational and professional topics - and a different teaching style to build up appropriate fundamental ideas and the justified use of mathematical concepts in intra- and extra-mathematical contexts might be a better choice for teaching and learning mathematics in full-time vocational education.

3 Geometry and algebra: Two cases in general versus vocational education

3.1 Geometry in general and vocational education: The case of the Pythagorean theorem

In order to illustrate differences between general and vocational education, we briefly sketch how the Pythagorean theorem (and related topics) are taught in both types of schools and colleges. In *general education* - at least in *Realschule* and *Gymnasium* - the theorems on squares on the sides of a right triangle (the Pythagorean theorem and Euclid's theorem) are often taught as an opportunity to study mathematical proofs and logical relations of properties - such as the inversion of hypothesis and conclusion. The theorems are then used to construct segments of a prescribed length or to algebraically calculate lengths of segments (also as an illustration of or introduction to square roots). Students (aged between 13 and 16 years) normally find this topic rather difficult and basically recall it as the problem of finding the appropriate right triangle in a complicated figure in order to calculate the length of a line segment. For the students, the problem of identifying this triangle and sometimes of transforming one of the formulas is the fundamental difficulty in this topic area. Normally, the proofs are not recalled by the students (for details see Andelfinger 1988, pp. 228-233).

Especially in technical (part-time) *vocational colleges*, how the Pythagorean theorem is taught changes drastically. Aspects of deduction and logic - the proofs- totally disappear, as well as the identity of the areas of the square on a leg of the right angle and the rectangle on the hypotenuse, or Euclid's theorem. Consequently, innerdisciplinary links vanish. Only the well-known Pythagorean identity is taught. As a "proof", a majority of textbooks offer only the identity 32 + 42 = 52 and an illustration (the well-known Pythagorean figure of a right triangle with sides of length 3, 4 and 5 and the hypotenuse in a horizontal position). Students have to numerically or algebraically calculate lengths of line segments, sometimes avoiding transformation of formulas by rote learning of all equivalent identities. Teaching the Pythagorean theorem in technical and vocational colleges -at least in the majority of classes- is thus reduced to the algorithmic, tool aspect of mathematics. In general, business and administration classes totally omit this topic from geometry.

3.2 Equations - formulas - algebra: Differences and common features within vocational education

Teaching equations in *part-time vocational colleges* is still dominated by traditional concepts of variables; that is, variables are seen as "unknowns" (e.g., in x2-8x+7=0), as "general numbers" in algebra (e.g., in (a+b).(a-b)=a2-b2), as quantities in formulas (e.g., in Ohm's law, V=R·I) and as function variables (e.g., in y=x2). Common properties as well as differences in these uses of variables are normally not mentioned.

A typical example from a textbook used in part-time vocational courses explicitly speaks of *the* unknown even if an equation has more than one solution - leading to an incomplete, incorrect solution:

(x-4)2=9
x-4=3
x=7

Transformation of equations and formulas are often reduced to a schematic procedure, a recipe with prescriptions like "plus will be minus on the other side of the equation." Only one teaching aid is sometimes used in these courses, namely a scale beam (and a pair of balance pans). Restrictions in the treatment of equations stemming from this type of aid (e.g.: the pair of scales makes sense only with equations having a unique positive solution; only a reduced part of the transformations can be represented) are not mentioned. For the majority of teachers and courses in part-time vocational education, other teaching aids seem to be unknown. Specific helps exist for the transformation of frequently used and simple formulas.

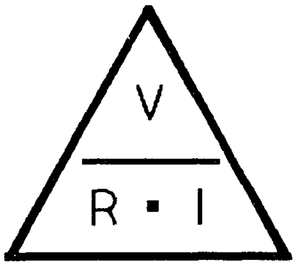

Fig.1: "Ohm's triangle"

As an example, Figure 1 shows "Ohm's triangle" for the formula $V = R \cdot I$ from electricity: hiding the variable to be calculated displays the algorithm to calculate that variable. This special device may help students remember all transformations of Ohm's law, "avoiding" rote learning of all three "different" formulas, but it will not develop algebraic understanding.

In *full-time vocational and technical colleges*, a "modern" concept of variable and equation may be found (introduced by an imitation of the new-math movement of the 1970's). We find "variables", "equivalent terms", and "equivalent transformations" of equations as well as formal notation from set theory and mathematical logic. Sometimes the correct description of an equation's solution set is very strictly prescribed (i.e., reduced to literally one acceptable expression). Statements like "The equation 2x=6 has the (one and only) solution x=3" could even possibly be characterised by the teacher or the textbook as incorrect.

Surprisingly, perhaps, the teaching of algebra in both part-time and full-time vocational and technical colleges thus has one characteristic feature in common: For both types of colleges, algebra often consists of meaningless procedures, of mere algorithms for manipulating symbols, and no attention is paid to an intelligent use of variables, equations, and formulas that would be meaningful in certain contexts (for a variety of constructive hints for teaching algebra in full-time technical colleges, cf. Bardy et al. 1988, pp. 171-210).

4 Teachers' views of teaching mathematics that is integrated into vocational knowledge

In the light of the research cited above, it is essential to know how teachers, when teaching in vocational and technical colleges, handle the interplay of the knowledge related to the vocation for which the students are being trained (*vocational knowledge*) and the mathematical knowledge. How do they manage to cope with the fact that the disciplinary structure of mathematics is usually incongruent with the vocational needs that sometimes are codified in nonsystematic ways, developing according to experience in the workplace? As a preliminary answer to this question, we analyse the way teachers of vocational and technical colleges think of the relationship between mathematical and vocational knowledge (for details see Strässer/Bromme 1989).

In 1981 and 1983, 40 vocational colleges teachers who taught a whole range of vocations (from business, administration to electricity and tailoring) were interviewed to learn about their conceptions of the relation of mathematics to the corresponding vocational domain. These teachers were trained in the vocational domain (e.g., metalwork or business) and then became teachers of vocational mathematics in part-time vocational colleges. To develop a narrow relation to the interviewee's teaching reality, the interview started with a detailed description of a course they had taught during the last college term, the topics taught in the course and their teaching methodology. At the end of the interview, data were gathered on the interviewee's biography. A comparison of data on the courses and teachers interviewed with data available on vocational teachers and classes in West Germany shows that the sample represented "average" vocational teachers (employed on a full-time basis). They taught courses to students holding relatively good school leaving certificates (for details cf. Strässer 1982, p. 60ff).

The 6 female and 34 male teachers (their average age was 42) had (with two exceptions) gone through preservice teacher training. Sixteen of them mentioned having had special studies in mathematics during their teacher training. They had an average of seven years of active teaching. They had been teaching courses in business and administration (14 teachers), technical domains (22 teachers) and other subjects (4 teachers, e.g., courses for future florists).

The teachers' perception of the relation between mathematical and vocational knowledge was explicated by a content analysis. Two independent raters examined the interview transcripts to identify those parts directly referring to the *function* mathematics had in the students' vocational training. These passages were classified into mathematics (a) for communication purposes, (b) as a tool, (c) as a description of a vocational situation and (d) in other functions. In a second step, the raters identified passages directly referring to the *relationship* between mathematical and vocational knowledge, analysing these passages by classifying their degree of abstractness and how mathematical and vocational knowledge were described interactively (e.g. mathematics taught before vs. after the related vocational knowledge).

Only half of the teachers remembered having taught mathematics for more than four consecutive lessons in isolation from vocational contexts. Fewer than half of the topics of the vocational arithmetic lessons (*Fachrechnen*) could be clearly labelled mathematics. The teachers gave examples like "the rule of three" or "equations", as well as "torque and force" or "calculating investment" as topics of their lessons. Nearly all of the teachers described their teaching method as presented in Section 2.1.

To learn about the three domains in the teachers' professional knowledge (i.e. pedagogy, mathematics, vocation), they were asked to distribute 100 points across three self-descriptions: "educator", "expert in mathematics", and "expert in the vocational domain for which they are trained". The distribution for 37 teachers is shown in Figure 2. As can be seen, the teachers' self-concept was that of being an educator (with a median of 40 points) or an expert in the vocational domain (median of 33 points), rather than an expert in mathematics (median at 10 points).

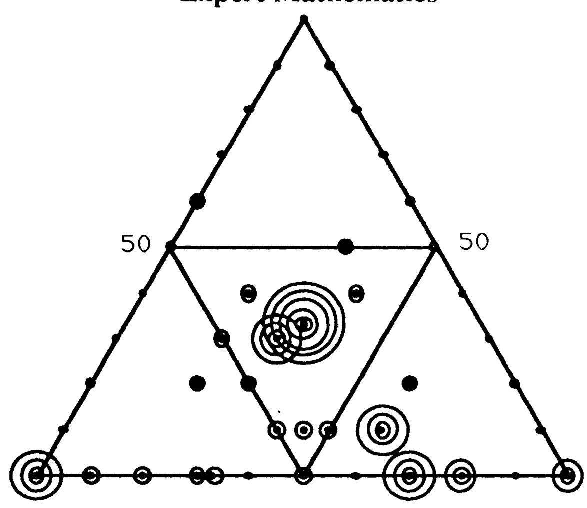

Fig. 2: Teachers' Self-Description

(Figure 2 is a "structogram" showing the distribution of 100 points by means of barycentric coordinates. Every circle around a point stands for one teacher's distribution of his/hers 100 self-description points according to the three coordinates.)

As for the functions of mathematics, half of the teachers considered mathematics to be a helpful tool for vocational contexts and nothing else. A third of them also mentioned the role of mathematics for understanding vocational situations. "Fostering logical thinking" and "general education" are examples of other functions of mathematics mentioned in the interviews.A detailed analysis of passages relating mathematics and vocational contexts reveals that the vast majority of the teachers thought of the relation between these two domains in terms of examples from both, where mathematics enhances vocational knowledge.

Statistical tests showed no significant connection between the teachers' self-description and their views on the function of mathematics for vocational contexts or on the relationship between mathematical and vocational knowledge. Teachers who had mathematical studies tended to give significantly more points for the "expert in mathematics" than the others. Teachers who had more years of active teaching seemed to put more stress on the understanding function than those with less teaching experience.

An interpretation of these results should concentrate on the fact that the teachers perceive only the tool and the understanding functions of mathematics. This more-or-less pragmatic view of mathematics also comes up in the widespread use of examples rather than whole disciplines or subjects when the relationship between mathematical and vocational knowledge was mentioned in the interviews. The fundamental purpose of mathematics is to serve as an operative tool when dealing with vocational problems. Its teaching is integrated into vocational contexts and has nearly no role in itself.

It might be surprising, perhaps, that no other conceptions of the relationship between vocational and mathematical knowledge were found in the interviews. An "objective" interpretation might assume that the above-mentioned conception was so dominant and widespread that different views could only rarely be found in vocational colleges. A "subjective" interpretation might mention the difficulties of speaking about the relation between mathematical and vocational knowledge. Since the teachers were not trained specifically in mathematics or mathematics education and had not been offered a clear conception of the use of mathematics and the relationship between mathematics and the real world, the obvious conceptual deficit does not come as a surprise.

5. Conclusion

Sections 1 to 4 clearly show the dichotomy underlying mathematics teaching and learning in vocational and technical colleges in Germany (cf. also Dehnbostel): Part-time colleges stress the tool aspect of mathematics by showing its use in a special vocational domain. This is a fundamental, indisputable aspect of mathematics teaching and learning which is integrated in vocational training. Nevertheless, a mere tool approach hides the social and cultural potential of mathematics, impeding professional advancement and a realistic view of the way mathematics is used in society (for a detailed discussion cf. Strässer/Barr/Evans/Wolf 1989)

In contrast to part-time vocational training and education, full-time colleges tend to imitate higher general education and -as far as possible- tend to pursue cultural, disciplinary and formal aspects of mathematics. In reality and because of time constraints, teaching and learning in these colleges seems to reduce these aims to a mere manipulating concepts from higher mathematics (e.g. calculus) by inculcating schemas and algorithms - leading to qualifications obviously devaluated by the growing use of computers and appropriate software.

Thus, all kinds of vocational colleges quite clearly reveal a common feature frequently found in mathematics classrooms in *all* types of schools: the reduction of mathematics learning to the learning of mere schemata and algorithms that can easily be applied by students in suitable tests and examinations without understanding.

Part-time as well as full-time vocational colleges are faced with the problem of teaching mathematics so that it can easily be used in a vocation or profession. Both have to cope with the problem of teaching mathematics that is imbedded and used in defined contexts while not impeding the application of the discipline to a broader range of situations. This complementarity might be developed by teaching basic ideas founded in reality (from algebra, geometry, calculus, and statistics, including a function concept to cope with relations and change) in order to use mathematics for intelligently modelling relevant situations from inside and outside the discipline.

An evaluation of this type of teaching and learning mathematics for vocational and professional colleges must not lose sight of difficulties linked to this aim: Some of the restrictions are due to the organisation of the human mind, which obviously tends to store knowledge only as relevant to the context it is learned in. Additional restrictions come from the way teaching is organised in the classroom, especially from the widespread compartmentalization and linearity of the teaching process. We do not see an easy solution to these contradictory constraints.