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PAPER

Tunneling time in attosecond experiment for hydrogen atom

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Abstract

Tunneling and tunneling time are hot debated and very interesting issues because of their fundamental role in the quantum mechanics. The measurement of the tunneling time in today's attosecond and strong field (low-frequency) experiments, despite its controversial discussion offers a fruitful opportunity to understand the time measurement and the role of time in quantum mechanics. In previous work Kullie (2015 *Phys. Rev. A* **92**, 052118), we suggested a model and derived a simple relation to calculate the tunneling time, which showed a good agreement with the experimental result for He-atom. In the present work we analyze and discuss our model considering the experimental result for H-atom, which is obtained recently by Satya Sainadh *et al* (2017 arXiv:1707.05445). In the tunneling region, we find that our model shows a good agreement with their experimental result (similar to the He-atom in our previous work), and with the accompanied time-dependent Schrödiger equation simulations. However, Sainadh *et al* use a different picture of the tunneling, in which tunneling time becomes an imaginary quantity. Whereas our model represents a real tunneling time picture, precisely a delay time with respect to the ionization time at the atomic field strength. Moreover, even though the above-threshold-ionization is beyond the tunneling regime, we still see that the actual ionization time is related to our model. However, crucial points arise and keep some questions open especially on the experimental side.

1. Introduction

The advent of attophysics opens new perspectives in the study of time resolved phenomena in atomic and molecular physics [1–4], the tunneling process and the tunneling time (T-time) in atoms and molecules [5–9]. Attosecond science (*asec* $= 10^{-18}$ s) concerns primarily electronic motion and energy transport on atomic scales and is of fundamental interest to the physics in general. In previous work [10, 11] we presented a tunneling model by exploiting the time-energy uncertainty relation (TEUR), precisely that time and energy are conjugate pair. The model has led to a nice relation to determine the T-time in a good agreement with the experimental finding in an attosecond experiment for He-atom (Keller Attosecond Experiment) [5, 12, 13]. Our model offers a real T-time picture and represents a delay time with respect to the ionization at atomic strength field F_a (compare figure 1). It is also interesting for the tunneling theory in general, because in this model the T-time is directly related to the height of the barrier [10, 11].

It is worthwhile to mention that Galapon [14–16] showed in a skillful mathematical way (the consistency theorems) that, there is no *a priori* reason to exclude the existence of a self-adjoint time operator canonically conjugate to a semibounded Hamiltonian, contrary to the (famous) claim of Pauli. Roughly speaking, see Garrison [17], for a canonically conjugate pair of operators of a Heisenberg type (i.e. uncertainty relation), Pauli theorem did not apply, unlike a pair of operators that form a Weyl pair (or Weyl system.) Indeed, since the appearance of quantum mechanics time was controversial, the famous example is the Bohr-Einstein weighing *photon box Gedanken experiment*, (see for example [18] p. 132). In [10] it was shown that our tunneling model (section 2) in the attosecond experiments [5] is intriguingly similar to this Gedanken experiment, where the

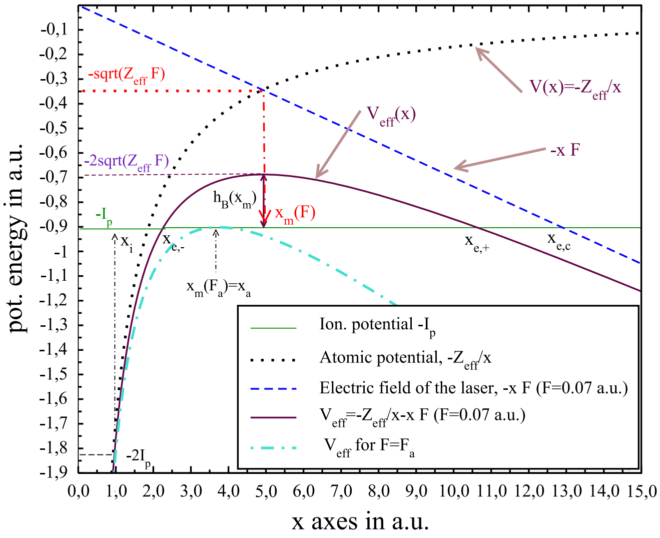

Figure 1. Graphic display of the atomic Coulomb potential $V(x)$, the peak electric field strength of the laser pulse F and the effective potential curves $V_{eff}(x)$, which form a tunneling barrier, above the ionization potential level I_p, with the two inner and outer points $x_{e,\pm} = (I_p \pm \delta_z)/2F$ and a barrier width $d_B = x_{e,+} - x_{e,-}$. Also shown the 'classical exit' point $x_{e,c} = I_p/F$ and the position at the maximum $x_m(F) = (Z_{eff}/F)^{1/2}$ of the barrier height $h_B(x_m)$, $x_a = x_m(F = F_a)$. The plot is for He-atom to show the role of Z_{eff} of a single-active-electron approximation. H-atom is similar with $Z_{eff} = 1$, $Ip = 0.5$ au.

former can be seen as a realization to the later. And as mentioned, the agreement of the model with the experimental result of [5], is impressively good, see [6, 10].

The T-time and the tunneling process itself in the attosecond experiments are hot debated and rather still not resolved completely. In the low-frequency attosecond experiments, the idea is to control the electronic motion by laser fields which are comparable in strength to the electric field in the atom. In today's experiments, usually used intensities are $\sim 10^{14}$ W cm^{-2}. For more details we refer to the tutorials [19, 20] (and the above mentioned.) A key quantity is the Keldysh parameter [9, 21],

$$\gamma_{_K} = \frac{\sqrt{2m_e I_p}}{eF}\omega_0 = \tau_K \omega_0, \tag{1}$$

where e, m_e are the charge and mass of the electron, I_p the ionization potential of the system (atom or molecule), ω_0 is the central circular frequency of the laser pulse or the laser wave packet and F, throughout this work, stands (unless it is clear) for *the peak electric field strength*, and τ_K denotes the Keldysh time. Hereafter in this work the atomic units are used, in which the electron's mass and charge and the Planck constant are set to unity, $\hbar = m_e = e = 1$. At values $\gamma_K > 1$ one expects predominantly photo-ionization or multiphoton ionization, while at $\gamma_{_K} < 1$ (field-) ionization happens by a tunneling process, which occurs for $F < F_a$, where F_a is the atomic field strength, see section 2. Tunneling happens for $F < F_a$, because the electron does not have enough energy to ionize directly. The electron tunnels (or tunnel-ionizes) through a barrier made by the Coulomb potential and the electric field of the laser pulse, and escapes at the exit point to the continuum as shown in figure 1 (a sketch for He-atom.)

In this paper we discuss our tunneling model considering the experimental result of Sainadh [22] for H-atom, and the accompanied theoretical result or the simulations of time-dependent Schrödinger equation (TDSE). In the attosecond experiments, precisely attosecond angular streaking experiments, the time is mapped to angular momentum using the rotating electric field vector of a nearly-circularly (elliptically) polarized laser pulse as streaking field. Where, in contrast to pump-probe experiments, a single pulse provides both the ionizing radiation and the streaking field. Depending on the time of ionization electrons are deflected in the angular spatial direction, so that the instant of ionization is mapped to the final angle of the momentum vector in the polarization plane. The very significant non-linearity of tunneling ionization ensures that the ionization rate peaks when the field reaches its maximum [22]. The measured offset angle is used to extract tunneling time, the procedure includes different contribution such as Coulomb correction θ_{Coul} and the streaking angle θ_{str}, for more details see [5]. In the experiment of Sainaddh *et al* [22] for H-atom, a short (few-cycle, 6 femtosecond duration) strong laser laser pulse is used, with a wave length $\lambda = 770$ nm and ellipticity of 0.84 ± 0.01. The intensity of the laser pulses was varied from $1.65 \cdot 10^4$ W cm^{-2} to $3.9 \cdot 10^4$ W cm^{-2} or peak field strengths $\approx 0.052 - 0.08$ au. At lower peak strengths ($F < 0.059$ the Keldysh parameter is $\gamma \sim 1$. However, in this region for long wave laser, tunneling process is usually recognized and the contribution form multiphoton ionization are or assumed to be small. For pulses used in the experiment is inside a region called 'Tunnel Oasis' by Reiss

[23], where according to Reiss tunneling models can be applied successfully without concern for the broader limitations of the tunneling approximation.

2. The tunneling time

Usually the tunneling process in the attosecond experiments is explained by a simple picture, like the one shown in figure 1 for the He-atom. It is based on the strong field approximation (SFA) or Keldysh-Faisal-Reiss approximation [21, 24, 25]; for introductory reviews see [9, 26, 27]. This simple picture is very useful in explaining the experiment, although it is strictly true only in the length gauge (see [23, 28, 29].) In the attosecond tunneling experiments, according to the SFA (see also [30]), tunneling means that the electron tunnels and escapes the barrier region at the exit point $x_{e,+}$ (see figure 1) with approximately zero kinetic energy. More precisely, the electron velocity along the (opposite) field direction is zero and negligible in other directions, also called a longitudinal fields approximation because the transverse fields are neglected [23]. In [10] we showed that the uncertainty in the energy, which is directly related to the height of the barrier $h_B(x_m)$, can be quantitatively discerned from the atomic potential energy at the exit point, $\Delta E \sim |V(x_e)| = |-Z_{eff}/x_e|$ for arbitrary strengths $F \leqslant F_a$, where Z_{eff} is the nuclear effective charge and the atomic field strength is given by $F_a = I_p^2/(4Z_{eff})$ [31, 32]. With the TEUR, $\Delta E \cdot \Delta T \geqslant 1/2$, one obtains the symmetrical (or total) T-time [10]:

$$\tau_{T,sym} = \frac{1}{2}\left(\frac{1}{(Ip+\delta_z)} + \frac{1}{(Ip-\delta_z)}\right) = \frac{I_p}{4Z_{eff}F} \tag{2}$$

where $\delta_z = \sqrt{I_p^2 - 4Z_{eff}F}$ for a single active electron model, and for the H-atom $Z_{eff} = 1$, $\delta_z \equiv \delta = \sqrt{I_p^2 - 4F}$. We call equation (2) the symmetrical T-time or the total time, because it was obtained by a symmetrization procedure (similar to the Aharonov time operator [33, 34]) from the unsympathized T-time [10]

$$\tau_{T,unsy} = \frac{1}{(Ip-\delta_z)} \tag{3}$$

The relation in equation (2), besides the mathematical simplicity, aids a conceptual reasoning [10, 11]. The physical reasoning of this relation is the following: the barrier itself causes a delaying time $\tau_{T,d}$, where

$$\tau_{T,d} = \frac{1}{2(Ip-\delta_z)}, \text{ and} \tau_{T,i} = \frac{1}{2(I_p+\delta_z)} \tag{4}$$

$\tau_{T,d}$ is the time delay with respect to the ionization at atomic field strength F_a, where the barrier is absent (the barrier height, the barrier width $d_B = \delta_z/F$ and δ_z are zero). It is the time duration for a particle to pass the barrier region (between $x_{e,-}$, $x_{e,+}$) and escapes at the exit point $x_{e,+}$ to the continuum [10]. The first term $\tau_{T,i}$ in equation (2) is the time needed to reach the entrance point $x_{e,-}$ from the initial point x_i, compare figure 1. The two steps of the model coincide at the limit $F \to F_a (\delta_z \to 0)$, and the total time becomes the ionization time $\tau_{T,sym} = \frac{1}{Ip}$, or $\tau_{T,d} = \tau_{T,i} = \frac{1}{2Ip}$ at the atomic field strength F_a. For $F > F_a$ the barrier suppression ionization (BSI) sets up [35, 36]. At the opposite side of the limit $F \to 0$, $\delta_z \to I_p$ and $\tau_{T,d} \to \infty$, hence nothing happens, i.e. the electron remains in its ground state undisturbed, which shows that our model is consistent, for details see [6, 10, 11]. It is worthwhile to mention that Chang *et al* [37] found a better estimation of the atomic field strength, it is slightly larger than the estimation of Augst *et al* [31] ($F_a^{Cha} \geqslant \sim Fa^{Aug}$), which is used in our work. The difference leads to a slightly larger δ_z values and hence slightly larger T-time $\tau_{T,d}$ equation (4). The work of Chang *et al* concerns only rare gas atoms, the effect, however, is too small and our model and conclusion will not change.

3. The Hydrogen atom

As mentioned above $\tau_{T,d}$ in equation (4) provides an excellent agreement [10] with the experimental result for He atom [5]. The issue is hot debated and controversial discussions still exist particularly for the T-time. Quantitatively due to the different models used to calculate the T-time and qualitatively whether the T-time is real, complex or imaginary quantity. Moreover, some authors rejected the use of the TEUR to derive our model [10], i.e. arguing that the T-time is an imaginary quantity, and an observable (time operator) does not exist for the time, hence the time is only a parameter in quantum mechanics. However, the later issue is rather resolved since Galapon [14–16] showed that there is no *a priori* reason to exclude the existence of self-adjoint time operators, as mentioned in section 1. See also [6] where a comprehensive discussion is given.

We discuss now our model equations (2)–(4) in regard to the experimental result of [22] for H-atom and the accompanied TDSE simulation. In figure 2 we plot the data shown in figure 3 of [22] together with our

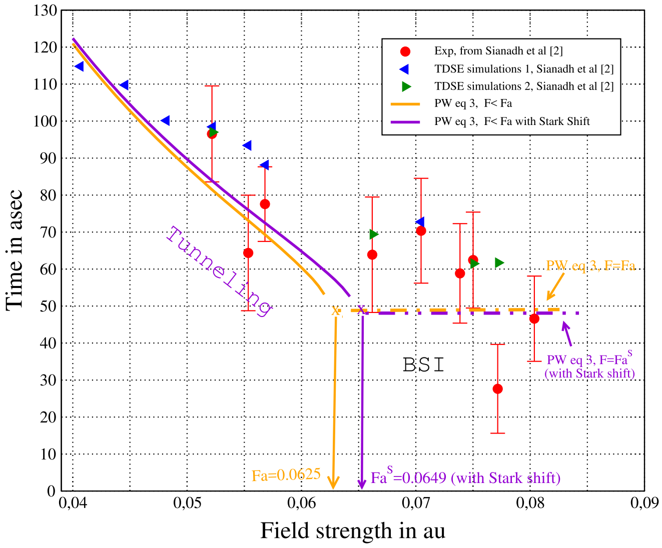

Figure 2. Time in attosecond versus field strength in atomic units for Hydrogen atom. Comparison of the unsymmetrical T-time equation (3) (with and without Stark shift) of the present work (PW), with the experimental result of [22] (red points, with the error bars), and TDSE simulation (triangle) of the same work [22]. The tunneling and BSI regions are separated by the atomic field strength without (F_a) and with Stark shift (F_a^S), see text.

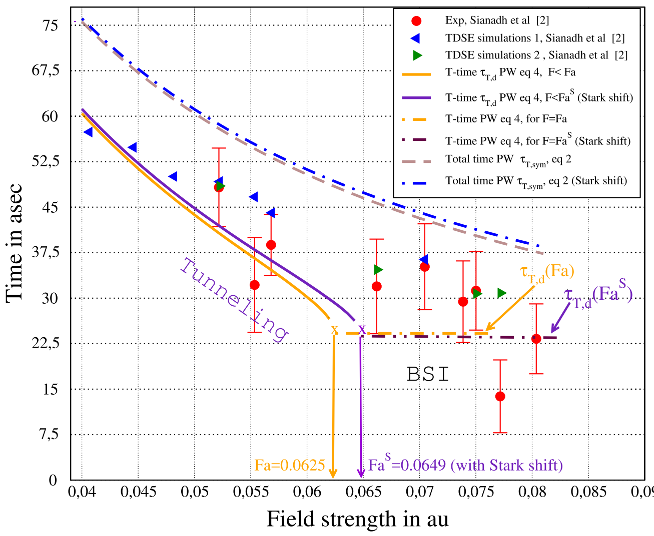

Figure 3. Time in attosecond versus field strength in atomic units for Hydrogen atom. Comparison of the T-time $\tau_{T,d}$ equation (4) (with and without Stark shift) of PW, with the experimental result of [22] (red points, with the error bars), and TDSE simulation (triangle) of the same work [22]. The tunneling and BSI regions are separated by the atomic field strength, without (F_a) and with Stark shift (F_a^S). Also shown total time $\tau_{T,sym}$ equation (2) (higher two curves), and the limit in equation (4) $\tau_{T,d}(F = F_a)$ and $\tau_{T,d}(F = F_a^S)$, compare equation (5), see text.

unsymmetrical T-time of equation (3). In the tunneling regime $F < F_a$ (for H-atom $F_a = 0.062\,5$au and with Stark shift $F_a^S = 0.064\,89$ au), we see a good agreement. Nevertheless, important points have to be clarified.

First for the conversion of the measured angle θ to a time variable, Sainadh *et al* used the factor 1° is equivalent to 7.13 *asec* $\approx$ $2600/360^\circ$ (2600 *asec* $\approx$ a period of the 770 nm pulse [22]). One notes that an angle is a pure number and cannot define an operator to represent an observable. In this result a factor $1/2$ is absent (unlike [5] for He-atom), which was recovered by the symmetry consideration in our model [10], section 2. We re-evaluate the data of Sainadh *et al* considering this factor and plot the result again in figure 3, and as seen the overall picture is the same. Again, we see in figure 3 a good agreement in the tunneling region (separated by the atomic field strength) between the two results of Kullie and Sainadh *et al*, but now $\tau_{T,d}$ of equation (4) is used, which is the actual T-time for $F < F_a$, as mentioned in section 2.

Second for $F > F_a$ there is no tunnel-ionization, because starting at the atomic field strength, see figure 1, there is no barrier and the electron is directly ionized. Following the consideration of Kiyan [36], for such a short pulse used in the experiment, the ionized electrons for $F > F_a$ are mostly due the BSI.

One should refer to it as the ionization time of a BSI process, which is classically a allowed ionization process [35]. On the basis of the present picture, the TDSE simulation do not make a distinction between tunneling (T-time) and ionization (ionization time), which is qualitatively a crucial point for two reasons. The way the time variable interning in TDSE, where some authors claim that only this time is a parameter and not generally the time in quantum mechanics [38, 39]. And second, in the model used in the TDSE simulation by Saindadh *et al* [22] and others, e.g. [40] (see also [41]), the T-time is claimed to be an imaginary quantity (though tunneling is a physical process). Whereas the real time (of the measurement or calculated by the simulations) is attributed to the tail of the Coulomb potential (after the exit point), i.e. when the tunneling, the so-called under-the-barrier process, is over. It worthwhile to mention that the introduction of an imaginary time is not a binding reasoning for tunneling methods [41, 42]. The imaginary T-time point of view relies on the assumption that, the T-time can be defined relatively to the case of a short-range potential, for which the T-time was found to be zero by using a computational model. For a system of short-range potential the atomic field strength is too small $F_a^{sr} = (I_p^{sr})^2/(4Z_{eff}) \approx (2I_p^{sr})^{3/2}/16$, but still a barrier can exist for $F < F_a^{sr}$. An example is the Hydrogen anion H^-, which has a single bound state $I_p^{sr} \approx 0.754\ 2$ eV ≈ 0.028 au and $F_a^{sr} \approx 0.000\ 8$ au. In our view to fix the ionization potential I_p and replace the Coulomb potential with Yukawa potential (as usually done) and at the same time hold (some) other parameters of the system unchanged, is questionable. Hence a numerically calculated time using a constructed model for short-range potential does not tell much about T-time or to conclude that the T-time is generally (or ultimately) an imaginary quantity. Apart from the fact that instantaneous tunneling (imaginary T-time picture) is limited by the special relativity because the particle (an electron or a photon) being moved (tunneled) instantaneously through a potential barrier leads to superluminal velocities [5, 43, 44]. Sainadh *et al* argued in [22] to rule out the attosecond range of T-time and claim that a finite T-time has to be explored in the zeptosecond ($zs = 10^{-21}$ s) time domain. A photon needs about $0.176 - 3.53$ *as* to transverse a typical barrier of the length $1. - 20.$ au. Thus, a T-time of a particle (electron or photon) in the zs-range is superluminal, i.e. it cannot be related to the transport of energy or the traverse of a particle through a barrier, as widely discussed and explained by Winful [43, 44].

However, the two pictures which we have discussed, can be also seen quantitatively equivalent or belong to two physically equivalent pictures. But this has the consequence that the barrier region is not necessary captured solely by an imaginary time component. And a complex T-time (real and imaginary components), which is reliable from a quantum-mechanical point of view, will not change our conclusion of a real T-time component in attosecond range. Indeed, the discussion is still ongoing and both pictures of real or imaginary T-time offer a fruitful contribution. For a detailed discussion we kindly refer the reader to [6], and to [11, 45] in relation to various T-time definitions.

In our model the tunneling happens for $F < F_a$, where δ_z and $\tau_{T,d}$ are real quantities and although δ_z becomes imaginary for $F > F_a$, nevertheless the total time is real for arbitrary field strength, see equation (2) and figure 3 upper curves. As the measurement considers the T-time, it neglects the first step or $\tau_{T,i}$, as discussed in [6, 10].

For a laser wave packet with a peak field strength $F > F_a$, the ionization happens in a different way and the quasistatic field approximation (QSA) is hardly valid. The electron, starting from its ground state, first undergoes a collisions with the laser wave packet (the first step in our model), then it is ionized when the laser-field reaches the atomic field strength, say at $F_t \approx F_a$, i.e. before the maximum is reached at the peak of the pulse F_P. The electron is freed at $F_t \approx F_a$ and the ionization occurs at the front of the laser pulse [46, 47] (chap. 4, p. 80), or at the leading edge of the laser pulse [48], i.e. the ionization process happens when $F_t \approx F_a < F_P$ at attosecond time scale. A simulation by Lein [49] indicated that at high field strengths the ionization occurs preferentially before the peak of the field.

The increase of the field strength (from $F_t \approx F_a$) leads to strong non-adiabatic effects which cannot be neglected anymore, apart from other effects such as the Carrier-Envelope phase (CEP) stabilization, see below. We think that is why for $F > F_a$ the measured or constructed time values of the experiment show an unexpected spread over a large interval of time, clearly seen in figures 2, 3. It can be interpreted that the ionization itself happens at $F_t \approx F_a$, where the mean value of the measurements follows the (straight) line $\tau(F_a) = \frac{1}{2I_p}$, as can be seen in figure 3 (or $\frac{1}{I_p}$ for figure 2). In other words for $F > F_a$ the ionization time can be approximated with

$$T_{F>F_a} = \tau(F_a) + \Delta t = \frac{1}{2I_p} + \Delta t \tag{5}$$

where Δt accounts for the contribution from the effects, which are beyond the QSA. A quick look to figure 3 shows that a better agreement can be reached between the τ_{F_a} and the experimental values by eliminating these effects in the experiment, or by extracting their contribution from the experimental data. Because at this limit $F \approx F_a$ the electron is freed, and no further contribution to the actual internal time of the ionization, and Δt is not attributed to the intrinsic time of the ionization. Despite that the calculations of Δt can be complicated, and its estimation experimentally is very difficult [4], but it can be seen as a perturbation to the ionization time τ_{F_a}, mostly due to the non-adiabatic effect (see below section 3). Apparently, the QSA loses its validity in the BSI region $F > F_a$, Nondipole effects becomes large, the dipole approximation breaks down [50], apart from a strong Stark effect (see below). This is unlike the tunneling region $F < F_a$, where the different effects do not have a noticeable contribution.

For $F \leqslant F_a$ one relies on the QSA, according to which the field strength F at the peak intensity is used. Because in the tunneling region the tunneling rate is maximized, when the field F reached its maximum. This picture is reasonable in interpreting the time measurement in attosecond experiment now for H-atom (previously for He-atom [6, 10, 11]). Whereas various effects become strong in the BSI region, i.e. for $F > F_a$ and leads to a complicated picture. During the classical motion in the BSI region, the electron can acquire some energy from the external electro-magnetic field. The value of this energy depends on the field phase when the electron leaves the barrier [35]. It has to be mentioned that for He-atom [5] the measurements were achieved only for $F < F_a$.

Finally in the tunneling region ($F < F_a$), we also see a good agreement between the TDSE simulation and our model. The exit point is the common starting point for both model, but it is defined in different ways. Usually a classical tunneling exit point $x_{e,c} = I_p/F$ is used in the TDSE simulation, whereas it is $x_{e,+}$ (figure 1) in our model, for details see [6, 11]). That could be the reason behind the differences between the two models in the tunneling region, compare figure 3, because the difference between $x_{e,c}$ and $x_{e,+}$ increases with F and is maximal at F_a. In fact, the barrier width in our model is smaller than the classical one, $d_B = \frac{\delta_z}{F} < d_c = I_p \Big/ F$, which makes the time to cross the barrier region smaller. As seen in figure 3 the TDSE simulation values are higher than our values, which is more pronounced when it comes closer to F_a. On the other hand, $d_B \approx d_c$ for small F ($\lim_{F\to 0} \delta_z = I_p$), and the difference between the two models is small, compare figure 3. One notice that $d_c = I_p/F$ is a vague definition of the tunneling region and can not offer a clear separation between of tunneling ($F < F_a$) and BSI region ($F > F_a$). Whereas it is clearly defined in our model, by F_a and the barrier width d_B and height $h_B(x_m)$ [6, 11]. The atomic potential energy at the exit point $x_{e,+}$ is related to the barrier height [10, 11] (unlike $x_{e,c}$), which is of great importance for the tunneling methods and for the tunneling theory in quantum physics. In addition, $x_{e,\pm}$, to be called 'enterexit', allows to consider the symmetry of the process or the system, since $x_{e,\pm} = (I_p \pm \delta_z)/(2F)$.

Moreover, in TDSE simulation one argues that the tail of the Coulomb potential (after the exit point) is responsible for the real time delay of the measurement, whereas the time under-the-barrier is claimed to be an imaginary quantity. This leads for $F \to 0,\ d_c \to \infty$ to the so-called Hartman effect, where the tunneling time becomes independent of the barrier length or it saturates with distance [43]. As for $d_c \to \infty$ the tail of the potential vanishes, no real time corresponds to a thick enough barrier. It is a consequence of the imaginary time picture(s), and as widely discussed by Winful [43, 44] it is not a transit time of a quantum particle. It is connected to the steady-state picture, which was criticized by Collin [51] and well discussed in [6]. In contrast, our model [10] benefits form the dynamical (intrinsic) time view [52–54] (chap 3). And unlike the velocity gauge picture (vector potential), in the length gauge picture as seen in figure 1 the electron is free at the exit point (in accordance with the SFA), the potential energy curve is bent down, the attraction of the nucleus is screened by the electric field, or the electric field acts like a charge, and screens the nucleus potential completely at $x_{e,+}$ in accordance with Einstein and the Feynman point of view that the fields and charges are physically not two different, independent entities, see Mead [55]. Nevertheless, the two models (TDSE simulation and our model) can be seen as two different, equivalent pictures of the same process. One has to mention that the length gauge, due to (Göppert-Mayer gauge-transformation [56], has the advantage to lead to mathematical expressions each of which have a ready physical interpretation [57].

For $F > F_a$ the different effects beyond the QSA should be considered to ensure a better agreement with the experimental values. However, an important point has to be mentioned here, we see that the trend of the TDSE simulation follows the trend of $\tau_{T,sym}$, although the former is below the later. As we discussed in [6], that is because in the TDSE simulation one neglects the first step, since the zero-point of the time t_0 is defined by the field direction when it reaches its maximum, where a critical comment can be found in the supplemental material of [8].

Stark shift In strong electromagnetic (AC) field, there is various effects unlike the case of a static electric (DC) field. The effects can strongly influence the laser-induced ionization process. Especially the non-adiabatic and depletion effects [58–61], and the Stark shift [46]. Sainadh *et al* [22] reported that in their experiment CEP was not stabilized (but argued that the issue is resolved by using elliptically polarized laser pulse), whereas the non-adiabatic and depletion effects were not reported. Further effects are the relativistic and the radiation pressure, and the breakdown of the dipole approximation [23, 62]. Reiss [23] discussed and presented a clear picture of the different regimes, how they depend on the laser parameters and where the different effects get their critical values. According to [23] one finds that the later effects are small for the parameters used in the experiment of [22]. However, all the above mentioned effects are small in the tunneling region or for $F \leqslant F_a$ in this case.

The AC Stark shift for small laser intensities (at low-frequency) can be considered as DC Stark shift [35, 46, 63, 64]. For the ground state in the low-frequency radiation, the AC Stark shift coincides to an adequate accuracy with the level shift of a DC field [46]. The effect of the Stark shift on the T-time ($F < F_a$) can be seen in figure 3 (and figure 2), where we plot the curves including the Stark shift perturbatively, which is a good approximation in this region $\delta E = -\frac{9}{4}F^2$ [46]. There is a small increase in the T-time and $F_a^S \approx 0.064\,89 > F_a = 0.062\,5$. The effect on the total time is smaller, as seen in figure 3.

For large field strengths, in the region $F > F_a$, the AC Stark effect according to [46] becomes dramatically different form the DC case, and the perturbation approximation loses its validity [35, 46, 64]. Apart from the AC Stark shift the non-adiabatic, the CEP and the depletion effects seem to be responsible for the spreading of experimental data in this region, but their investigation is beyond the frame of the present work. It is worth noting that, under extreme conditions, rigorously speaking for higher field strengths the concept of the discreteness of an atomic spectrum breaks down and the concept of the Stark shift loses its meaning [46]. And it goes without saying that, it is interesting to gather more data from the experiments to obtain a clear or more reliable picture.

4. Conclusion

In this work we have discussed the T-time of our model regarding the experimental result for H-atom and the accompanied TDSE simulations in [22]. We found a good agreement between our model and the results of [22] in the tunneling region, similar to the case of He-atom [10]. Important points have been addressed and discussed in comparison to the experimental result or to the TDSE simulations. This confirms our model and our real tunneling time picture becomes doubtless, especially because the existence of time operator and time as an observable in quantum mechanics are rather resolved [14–16]. However, the debate continues in regard to the controversial interpretation of the experimental data and the separation of the different regions of the tunnel-ionization and ionization. For $F > F_a$ the non-adiabatic, the depletion, the Carrier-Envelope phase and the Stark effect are strong. We argued that the actual (intrinsic) ionization time is related to our model, where the contribution of these effects can be treated as a perturbation term, although its calculation can be sophisticated. These effects together with the relativistic effects, the radiation pressure and the breakdown of the dipole approximation, strongly influence the field ionization or laser-induced ionizations. It needs future investigations and further attosecond experiments to get a clear picture.

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